Unit 5 Trigonometry

Review:

SOH	CAH	TOA
$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$	$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$	$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

Ex) Determine the value of *x* for each of the following.

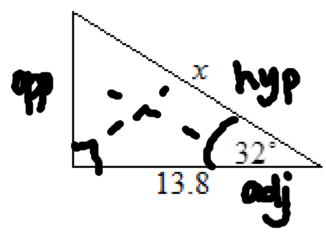
a)

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan 27° = \dfrac{x}{4.28}$

$\boxed{x = 2.18}$

b)



$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

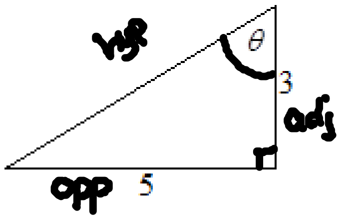
$\dfrac{\cos 32°}{1} = \dfrac{13.8}{x}$

$x = \dfrac{13.8}{\cos 32°}$

$\boxed{x = 16.27}$

Ex) Determine the value of angle θ in each of the following.

a)



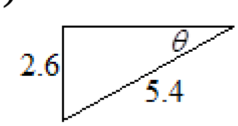
$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

$\tan\theta = \dfrac{5}{3}$

$\tan^{-1}(\cancel{\tan}\theta) = \tan^{-1}\left(\dfrac{5}{3}\right)$

$\theta = 59°$

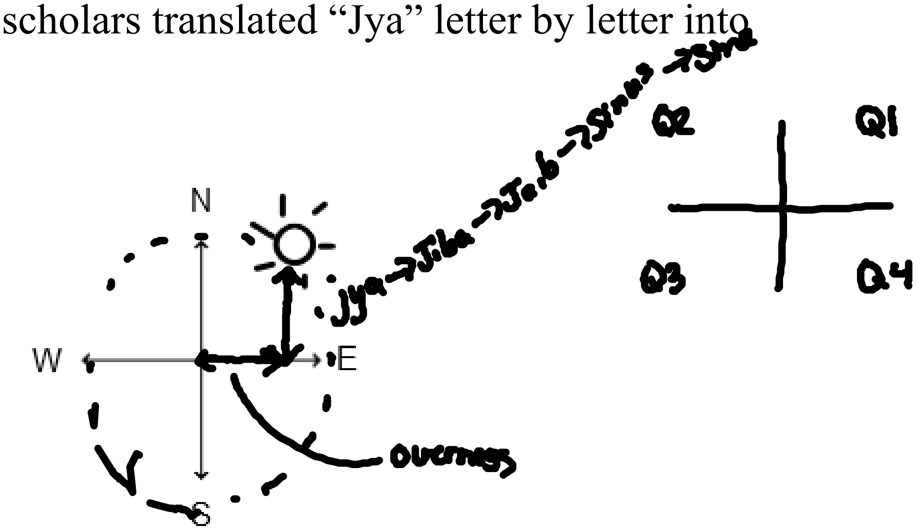
b)



$\theta = 28.8°$

The History of Trigonometry: How high is the sun?

- Humans wanted to determine the height of the sun at given time as it follows its circular path around the earth.
- The study of “circle-ometry” begins.
- 5th century: Indian scholars called the height of the sun the “Jya” in Sanskrit.
- 10th century: Islamic scholars translated “Jya” letter by letter into “Jiba” in Arabic.



- 12th century: European scholars translated jiba into Latin. Jiba was not a proper Arabic word so they thought it was a misspelled version of “jaib” which meant “a cove or bay” in Arabic. They translated this incorrect word into Latin for bay which is “sinus” which we later shortened to Sine!

Therefore, the Sine of the sun refers to its height.

- The horizontal displacement or “overness” was called the “companion length to sine” which was shortened to “**cosine**”.

Therefore, the cosine of the sun refers to its horizontal distance.

- 16 century: George Rheticus wrote about sine and cosine without mentioning circles, he used triangles. Circle-ometry becomes trigonometry, the study of right triangles.

Let's go back to the sun:

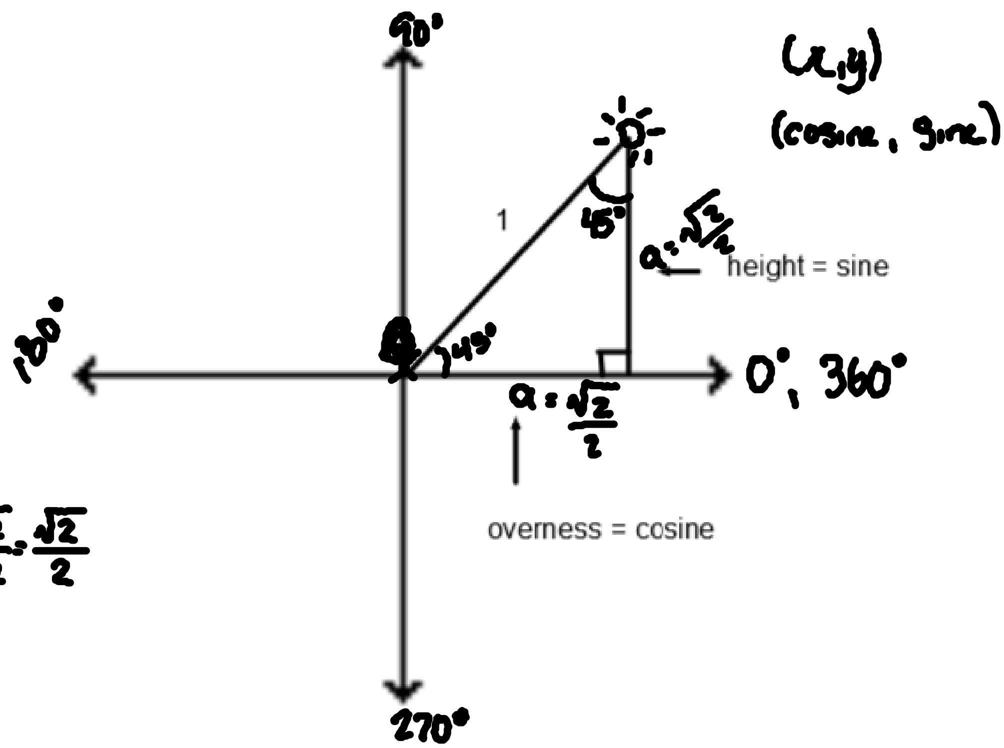
Use Pythagorean,

$a^2 + a^2 = 1^2$

$2a^2 = 1$

$a^2 = \frac{1}{2}$

$a = \sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



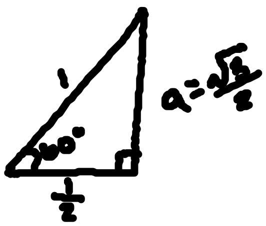
- What is the radius of the circular path? The distance from Earth to the sun…Astronomers called it 1 astronomical unit so we call the radius 1.

  What is height of the sun at 45^o? $\frac{\sqrt{2}}{2}$

  What is the overness of the sun at 45^o? $\frac{\sqrt{2}}{2}$

  Therefore,
  $\sin(45^o) = \frac{\sqrt{2}}{2}$ $\qquad\qquad$ $\cos(45^o) = \frac{\sqrt{2}}{2}$

"height at 45°"



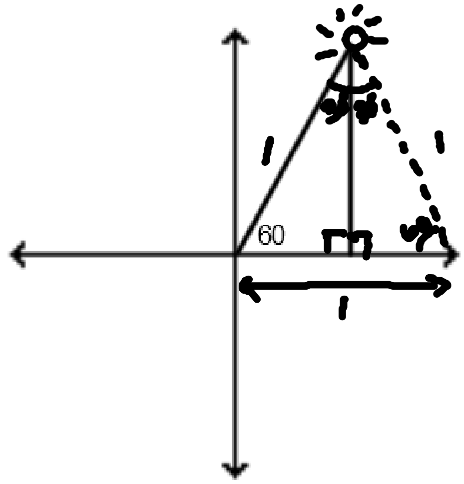
Pythagorean.

$$a^2+\left(\frac{1}{2}\right)^2=1^2$$

$$a^2=1-\frac{1}{4}$$

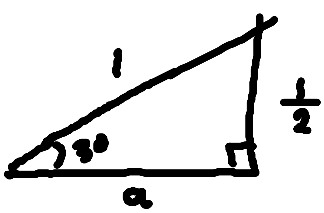
$$a^2=\frac{3}{4}$$

$$a=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{\sqrt{4}}=\frac{\sqrt{3}}{2}$$



What is the height of the sun at 60^o, $\sin(60^o)$? $\frac{\sqrt{3}}{2}$

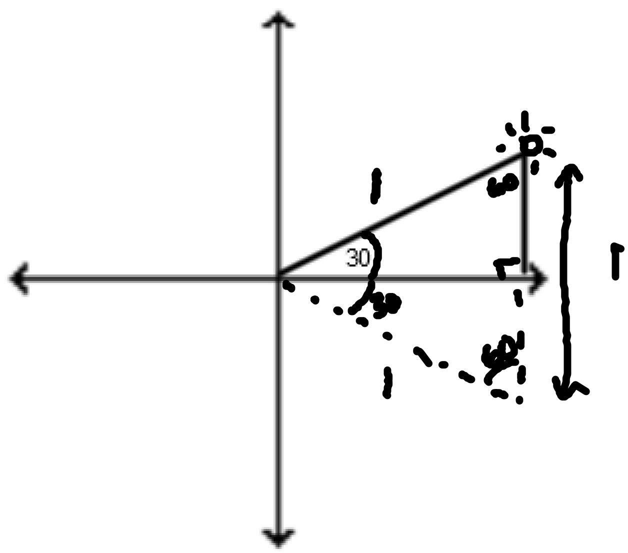
What is the overness of the sun at 60^o, $\cos(60^o)$? $\frac{1}{2}$



Pythagorean.

$$a^2+\left(\frac{1}{2}\right)^2=1^2$$

$$a=\frac{\sqrt{3}}{2}$$



What is the height of the sun at 30^o, $\sin(30^o)$? $\frac{1}{2}$

What is the overness of the sun at 30^o, $\cos(30^o)$? $\frac{\sqrt{3}}{2}$

Summary

$\sin 30^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$
$\sin 45^\circ = \frac{\sqrt{2}}{2}$	$\cos 45^\circ = \frac{\sqrt{2}}{2}$
$\sin 60^\circ = \frac{\sqrt{3}}{2}$	$\cos 60^\circ = \frac{1}{2}$

$\tan 30^\circ = \frac{\sin 30}{\cos 30} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

$\tan 45^\circ = \frac{\sin 45}{\cos 45} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

$\tan 60^\circ = \frac{\sin 60}{\cos 60} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

What about when $\sin(135^o)$?

What about when $\sin(225^o)$?

What about when $\sin(120^o)$?

What about when $\sin(330^o)$?

What about when $\cos(135^o)$?

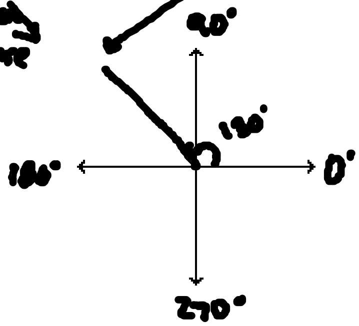
What about when $\cos(240^o)$?

Sketch the following angles.

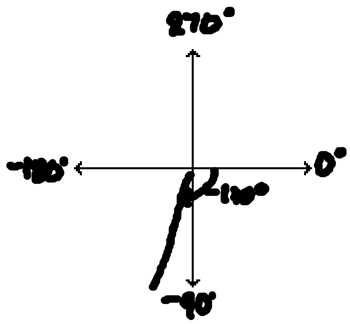
This is called terminal arm

130° will be in half Somewhere

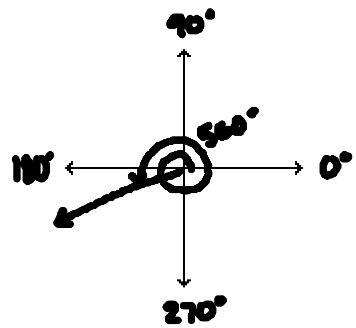
a) 130°



b) -110°



c) 560°



→ halfway would be 135° so 130° would be a little less than half way between.

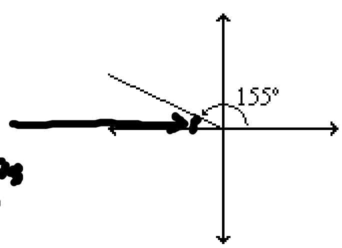
→ − means clockwise
→ We go backwards
→ −110 will be between −90° and −180° but closer to −90°.

→ 1 full rotation = 360°
→ 560° − 360° = 200°
→ We go around once and then 200° more

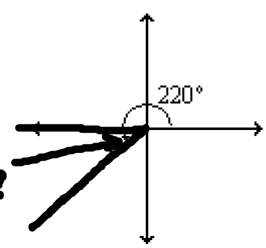
Reference Angles:

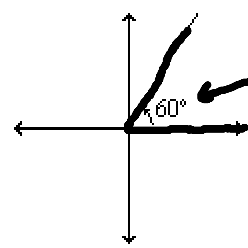
A reference angle is the angle between the terminal arm and the *x*-axis. Reference angles are always between 0° and 90°.

What angle is this?



what angle?





We want this angle.

→ We want the distance between x-axis and our terminal arm.

180° − 155° = 25°

x-axis

our arm is before axis

reference angle

→ 220° − 180° = 40°

our angle is past x-axis

x-axis

reference angle

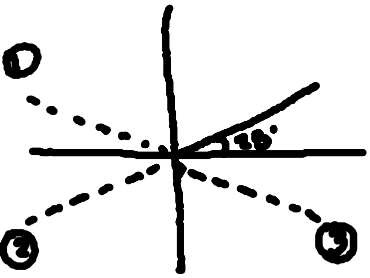
→ Reference angle = 60°

→ This is distance between x-axis (0°) and terminal arm

Ex) Determine the measure of the three other angles in standard position, $0^\circ < \theta < 360^\circ$, that have the same reference angle as

① Reference Angle $= 225^\circ - 180^\circ$
$= 45^\circ$

a) 28°

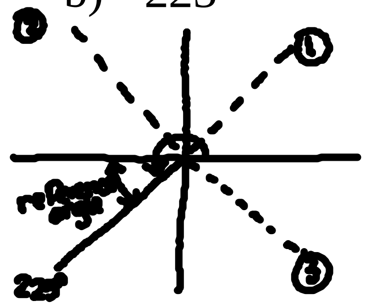


① $180^\circ - 28^\circ = 152^\circ$

② $180 + 28^\circ = 208^\circ$

③ $360 - 28^\circ = 332^\circ$

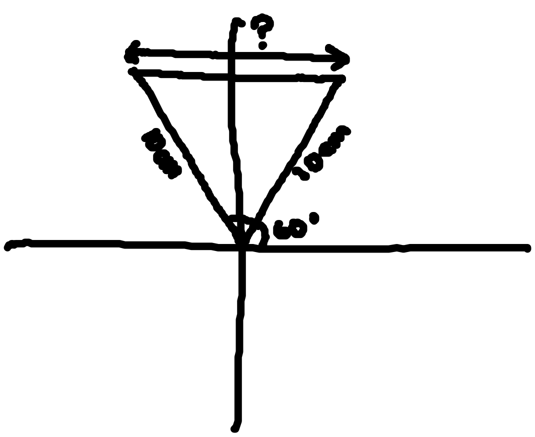
b) 225°



② $180^\circ - 45^\circ = 135^\circ$

③ $360 - 45^\circ = 315^\circ$

Ex) The pendulum arm of a metronome is 10 cm long. For one particular tempo, the setting results in the arm moving back and forth from a start position of 60° to 120°. What horizontal distance does the tip of the arm move in one beat?



IF I take just the Δ on the right...

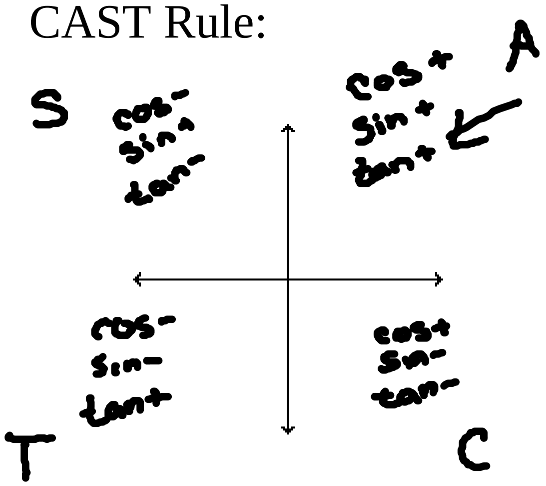
$\sin 30^\circ = \dfrac{opp}{10cm}$

opp = 5cm

→ If both triangles are the same the total distance is 10cm.

Now Try: Workspace Lesson 1

CAST Rule:



(cos , sin)

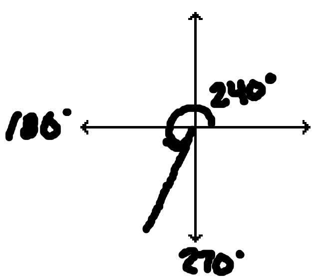
$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Ex) Determine the exact primary trigonometric ratios ($\sin\theta$, $\cos\theta$, $\tan\theta$) for each of the following.

Exact means fraction/radical.

a) 240°

Reference : 60°



$\sin 240° = -\frac{\sqrt{3}}{2}$

$\cos 240° = -\frac{1}{2}$

$\tan 240 = \frac{\sin 240}{\cos 240}$

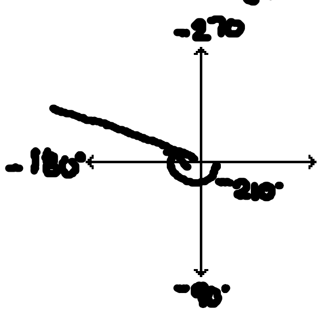
$= \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

$= -\frac{\sqrt{3}}{\cancel{2}} \times -\frac{\cancel{2}}{1}$

$= \sqrt{3}$

b) −210° clockwise

Reference = 30°



$\sin(-210°) = \frac{1}{2}$

$\cos(-210°) = -\frac{\sqrt{3}}{2}$

$\tan(-210°) = \frac{\sin(-210°)}{\cos(-210°)}$

$= \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}}$

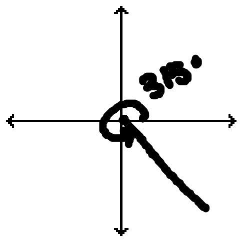
$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{-\sqrt{3}}$

$= -\frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{3}}$

c) 315°

Reference: 45°

315°



$\sin 315^\circ = -\frac{\sqrt{2}}{2}$

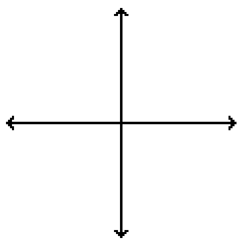
$\cos 315 = \frac{\sqrt{2}}{2}$

$\tan 315^\circ = \frac{\sin 315}{\cos 315^\circ}$

$= \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$= -1$

d) 150°



Reference $= 30^\circ$

$\sin 150 = \frac{1}{2}$

$\cos 150 = \frac{\sqrt{3}}{2}$

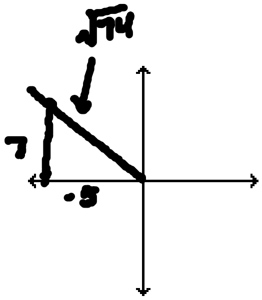
$\tan 150^\circ = -\frac{\sqrt{3}}{3}$

Ex) The point $(-5,\ 7)$ lies on the terminal arm of angle θ

.

$(\cos, \sin)$

Determine the three exact primary trigonometric ratios for θ.



Pythagorean

$(-5)^2 + (7)^2 = c^2$

$25 + 49 = c^2$

$74 = c^2$

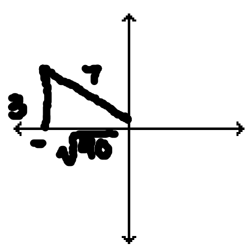
$c = \sqrt{74}$

$\sin\theta = \frac{opp}{hyp} = \frac{7}{\sqrt{74}} \times \frac{\sqrt{74}}{\sqrt{74}} = \frac{7\sqrt{74}}{74}$

$\cos\theta = \frac{adj}{hyp} = \frac{-5}{\sqrt{74}} = \frac{-5\sqrt{74}}{74}$

$\tan\theta = \frac{opp}{adj} = \frac{7}{-5}$

Ex) If $\sin\theta = \dfrac{3}{7}$ and $\cos\theta$ is negative, determine the exact primary trigonometric ratios for $\cos\theta$ and $\tan\theta$.



$\sin\theta = \frac{3}{7} = \frac{opp}{hyp}$

Pythagorean

$3^2 + b^2 = 7^2$

$b^2 = 7^2 - 3^2$

$b^2 = 49 - 9$

$b^2 = 40$

$b = \sqrt{40}$

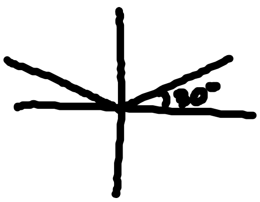
$\cos\theta = \frac{adj}{hyp} = \frac{-\sqrt{40}}{7}$

$\tan\theta = \frac{opp}{adj} = \frac{3}{-\sqrt{40}} \times \frac{-\sqrt{40}}{-\sqrt{40}} = \frac{-3\sqrt{40}}{40}$

Ex) Solve the following for θ.

a) $\sin\theta = \dfrac{1}{2}$, $0^\circ < \theta < 360^\circ$

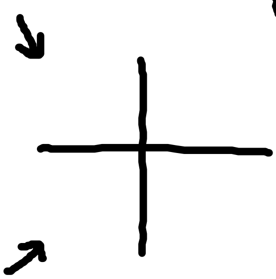
When is height of the sun $\frac{1}{2}$?



$\theta = 30^\circ, 150^\circ$

b) $\cos\theta = \dfrac{-\sqrt{2}}{2}$, $0^\circ < \theta < 360^\circ$

When is your overness $-\frac{\sqrt{2}}{2}$?



Ref $= 45^\circ$

$\boxed{\theta = 135^\circ, 225^\circ}$

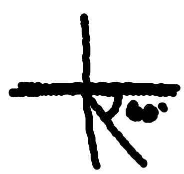
c) $\tan\theta = -\sqrt{3}$, $0^\circ < \theta < 360^\circ$

$\tan\theta = \frac{-\sqrt{3}}{1} = \frac{\sin}{\cos} =$

$\cancel{\tan^{-1}}(\cancel{\tan}\theta) = \tan^{-1}(-\sqrt{3})$

$\theta = -60^\circ$

$= 300^\circ, 120^\circ$



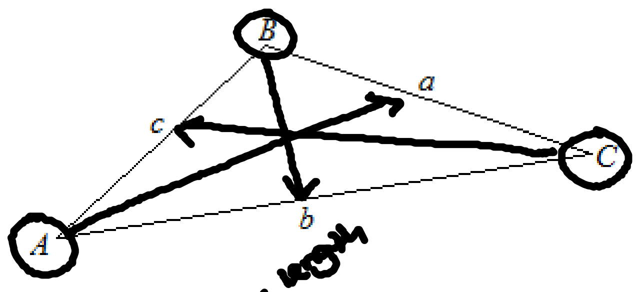
$360 - 60 = 300^\circ$

Now Try: Workspace Lesson 2

The Sine Law:

SOH CAH TOA only works on triangles that have a 90° angle. For triangles that are not right angle triangles we need a different way to solve for missing sides or angles. The Sine Law is one of these ways.

Angles in capitals
Side length are lower case.



side length

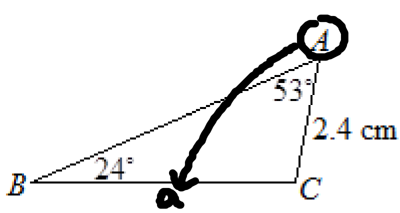
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

angle

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ex) Find the length of side *a* to the nearest tenth.



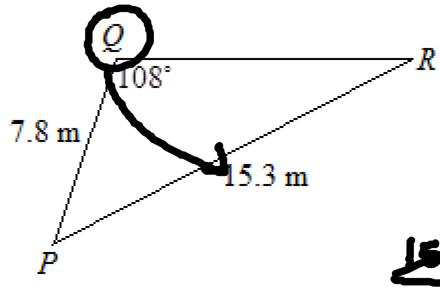
$$\frac{a}{\text{Sin}A} = \frac{b}{\text{Sin}B}$$

$$\frac{a}{\text{Sin }53} = \frac{2.4\text{cm}}{\sin 24°}$$

$$\frac{a\cancel{\sin 24°}}{\cancel{\text{Sin}24°}} = \frac{2.4\sin 53}{\sin 24°}$$

$a = 4.7$ cm

Ex) Find the measure of angle *R* to the nearest degree.



$\frac{q}{\sin Q} = \frac{r}{\sin R}$

$\frac{15.3\text{m}}{\sin 108^\circ} = \frac{7.8\text{m}}{\sin R}$

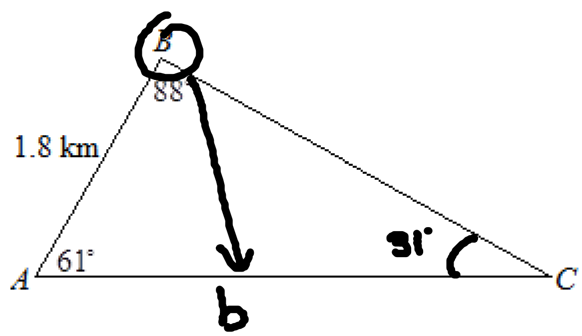
$\frac{\cancel{15.3}\sin R}{\cancel{15.3}} = \frac{7.8\sin 108^\circ}{15.3}$

$\sin R = \frac{7.8\sin 108^\circ}{15.3}$

$R = \sin^{-1}\left(\frac{7.8\sin 108}{15.3}\right)$

$=$

Ex) Determine the length of side *b* to the nearest tenth.



$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{b}{\sin 88^\circ} = \frac{1.8\text{Km}}{\sin 31^\circ}$

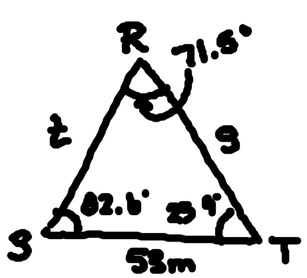
$b = 3.5$ Km

All angles add to 180 : $180^\circ - 61^\circ - 88^\circ = 31^\circ$

angle side

Ex) In ΔRST, $\angle S = 82.6^\circ$, $r = 53$ m, and $\angle T = 25.9^\circ$, Solve the triangle (find all missing sides and all missing angles). Round each side to the nearest metre.

① Angle : $\angle R = 180^\circ - 82.6^\circ - 25.9^\circ = 71.5^\circ$



② $\frac{t}{\sin T} = \frac{r}{\sin R}$

$\frac{t}{\sin 25.9^\circ} = \frac{53\text{m}}{\sin 71.5^\circ}$

$t = 24$m

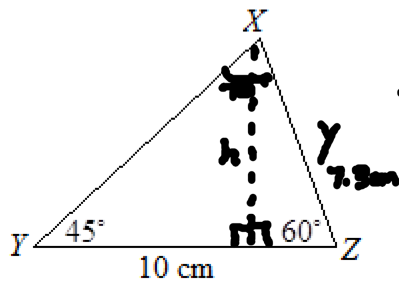
③ $\frac{s}{\sin S} = \frac{r}{\sin R}$

$\frac{s}{\sin 82.6} = \frac{53\text{m}}{\sin 71.5^\circ}$

$s = 55$ m

Ex) Find the area of triangle *XYZ* to the nearest unit.

$A = \frac{bh}{2}$



→ Want y.

→ $\angle X = 180 - 45° - 60 = 75°$

→ Side y

$\frac{y}{\sin Y} = \frac{x}{\sin X}$

$\frac{y}{\sin 45} = \frac{10cm}{\sin 75°}$

$y = 7.3cm$

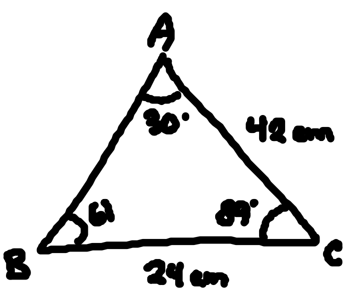
→ Find h · $\sin\theta = \frac{opp}{hyp}$

$\frac{\sin 60°}{1} = \frac{h}{7.3cm}$

$h = 6.3cm$

$A = \frac{bh}{2} = \frac{10cm \times 6.3cm}{2} = 31.5cm^2$

Ex) In ΔABC, $\angle A = 30^\circ$, $a = 24$ cm, and $b = 42$ cm, Solve the triangle (find all missing sides and all missing angles). Round all answers to the nearest unit.



$\angle C = 180° - 30° - 61° = 89°$

$\angle B$: $\frac{42cm}{\sin B} = \frac{24cm}{\sin 30}$

$\frac{42\sin 30}{24} = \frac{\cancel{24}\sin B}{\cancel{24}}$

$\sin^{-1}\left(\frac{42\sin 30°}{24}\right) = \cancel{\sin^{-1}}(\cancel{\sin} B)$

$B = 61°$

$\frac{c}{\sin 89°} = \frac{24cm}{\sin 30°}$

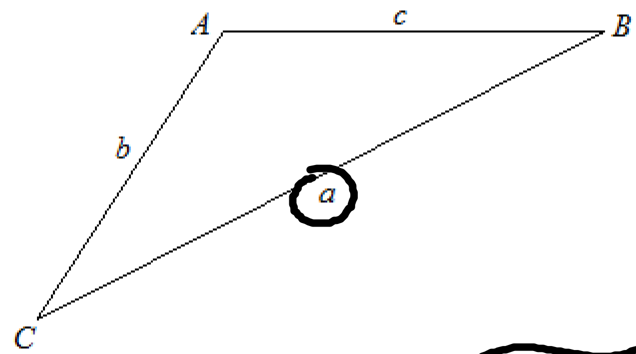
$\frac{c\cancel{\sin 30°}}{\cancel{\sin 30}} = \frac{24\sin 89°}{\sin 30°}$

$c = 48$ cm

Now Try **Page 108 #1 to 6, 8, 10** **11, 12, 21, 24**

<u>The Cosine Law</u>:

Again, like the Sine Law, the Cosine Law can be applied to all triangles.

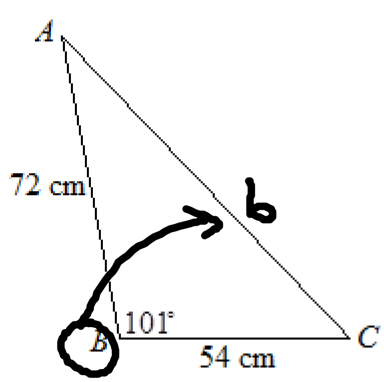


$$a^2 = b^2 + c^2 - 2bc\cos A \qquad\qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac\cos B \qquad \text{or} \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad\qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Ex) Find side b to the nearest cm.



$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = 54^2 + 72^2 - 2(54)(72)\cos 101^\circ$$

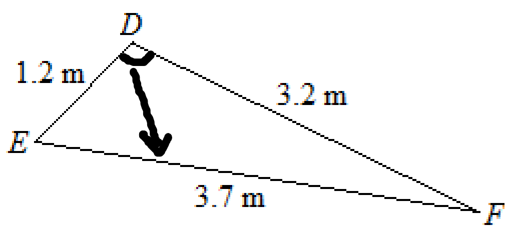
$$b^2 = 2916 + 5184 \underbrace{- 7776(-0.19)}$$

$$b^2 = 2916 + 5184 + 1477.44$$

$$\sqrt{b^2} = \sqrt{9577.44}$$

$$b = 98\text{ cm}$$

Ex) In ΔDEF, $d = 3.7$ m, $e = 3.2$ m, and $f = 1.2$ m. Find $\angle D$ to the nearest degree.



$$\cos D = \frac{1.2^2 + 3.2^2 - 3.7^2}{2(1.2)(3.2)}$$

$$\cos D = \frac{1.44 + 10.24 - 13.69}{7.68}$$

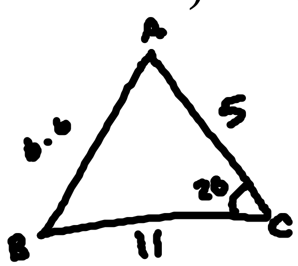
$$\cos D = \frac{-2.01}{7.68}$$

$$\cos D = -0.26.$$

$$D = \cos^{-1}(0.26..)$$

$$= 105^\circ$$

Ex) In ΔABC, $a = 11$, $b = 5$, and $\angle C = 20^\circ$. Solve the triangle.



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 11^2 + 5^2 - 2(11)(5)\cos 20^\circ$$

$$c^2 = 121 + 25 - 110(0.93)$$

$$\sqrt{c^2} = \sqrt{43.7}$$

$$c = 6.6$$

$$180^\circ - 20 - 34.7^\circ = 125.3$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{11}{\sin A} = \frac{6.6}{\sin 20}$$

$$\frac{\cancel{6.6}\sin A}{\cancel{6.6}} = \frac{11\sin 20}{6.6}$$

$$\sin A = \frac{11\sin 20}{6.6}$$

$$\sin^{-1}(\sin A) = \sin^{-1}\left(\frac{11\sin 20}{6.6}\right)$$

$$A = 34.7^\circ$$

Ex) A surveyor need to find the length of a swampy area. He sets up a transit at a point *A* and measures the distance from it to one end of the swamp as 692.6 m and the distance to the other end as 468.2 m. If the angle of sight between these two measurements is 78.6°, determine the length of the swamp.

Now Try
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