

Unit 4 Rational and Reciprocal Functions

Rational Expressions:

Rational expressions are fractions that have a polynomial numerator and denominator.

Ex) $\frac{2x-7}{x^2-9}$ $\frac{3}{x+2}$ $\frac{2y-4x}{5y+3x}$

↑ ↑ ↑ ↑

Any time a variable is found in a denominator the possibility of restrictions exist.

↓
You cannot divide by 0 - so no 0 in denominator

Ex) For each rational expression determine all non-permissible values.

↑
Values that your denomin. = 0

a) $\frac{5t}{4st^2}$

↑

$t \neq 0$
 $s \neq 0$

b) $\frac{3x}{x(2x-3)}$

↑

$x \neq 0$

$x \neq \frac{3}{2}$

△ $2x-3 \neq 0$

~~$\frac{x}{2} \neq \frac{3}{2}$~~

c) $\frac{2p-1}{p^2-p-12} = \frac{2p-1}{(p+3)(p-4)}$

$p+3 \neq 0$
 $p \neq -3$

$p-4 \neq 0$
 $p \neq 4$

Equivalent Rational Expressions:

Remember when the numerator and denominator of a fraction is multiplied or divided the same value the resulting fraction is equivalent.

$$\frac{1}{3} \stackrel{\times 2}{=} \frac{2}{6}$$

Ex) $\frac{7x}{x-2} \times \frac{2}{2} = \frac{14x}{2x-4}$

Simplifying Rational Expressions:

To simplify a rational expression you must first factor the numerator and denominator. Only like factors of the numerator and denominator will cancel each other out.

Ex) Simplify the following.

a) $\frac{x^3 y}{x^2 y^4}$ } both monomials
no factoring

$$= \frac{\cancel{x} \cancel{x} \cancel{x} y}{\cancel{x} \cancel{x} y y y}$$

$$= \frac{x}{y^3}$$

b) $\frac{3x-6}{2x^2+x-10} = \frac{3\cancel{(x-2)}}{(2x+5)\cancel{(x-2)}}$

$$= \frac{3}{(2x+5)}$$

$$x \neq -\frac{5}{2}$$

$$x \neq 2$$

c) $\frac{x^2 - 4x - 32}{x^2 + 9x + 20}$

$$\frac{(x-8)(x+4)}{(x+5)(x+4)}$$

$$= \frac{x-8}{x+5}$$

$$x \neq -5$$

$$x \neq -4$$

d) $\frac{1-t}{t^2-1}$

$$\frac{(1-t)}{(t+1)(t-1)}$$

$$= \frac{-(-1+t)}{(t+1)(t-1)}$$

$$t \neq -1$$

$$t \neq 1$$

$$= \frac{-\cancel{(t-1)}}{(t+1)\cancel{(t-1)}}$$

$$= \frac{-1}{t+1}$$

Ex) Consider the expression $\frac{16x^2 - 9y^2}{8x - 6y}$

a) What expression represents the non-permissible values of x ?

$$8x - 6y \neq 0$$

$$\frac{8x}{8} \neq \frac{6y}{8}$$

$$x \neq \frac{6y}{8}$$

b) Simplify the rational expression.

$$\frac{16x^2 - 9y^2}{8x - 6y} = \frac{(4x + 3y)\cancel{(4x - 3y)}}{2\cancel{(4x - 3y)}} \quad **$$
$$= \frac{4x + 3y}{2}$$

c) Evaluate the expression for $x = 2.6$ and $y = 1.2$

$$\frac{4(2.6) + 3(1.2)}{2}$$
$$= 7$$

Now Try

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9, 13, 14, 21, 26,

Multiplying and Dividing Rational Expressions:

Multiplying:

- Factor all numerators and denominators
- Simplify by eliminating common factors (1 from a numerator and 1 from a denominator)
- State all restrictions

Ex) Simplify the following.

$$\begin{array}{l} \text{Factor} \swarrow \\ \text{a) } \frac{x^2 - 3x - 10}{x^2 - x - 56} \times \frac{x+7}{x-5} \leftarrow \text{Factor} \\ \text{Factor} \searrow \end{array}$$

$$= \frac{(x-5)(x+2)}{(x-8)(x+7)} \times \frac{x+7}{x-5}$$

$$= \frac{\cancel{(x-5)}\cancel{(x+2)}\cancel{(x+7)}}{(x-8)\cancel{(x+7)}\cancel{(x-5)}}$$

$$\boxed{= \frac{x+2}{x-8}}$$

$$\begin{array}{l} x \neq 8 \\ x \neq -7 \\ x \neq 5 \end{array}$$

$$\text{b) } \frac{3}{a} \times \frac{a^2}{b}$$

$$= \frac{3a^2}{ab}$$

$$= \frac{3a}{b}, \quad \begin{array}{l} b \neq 0 \\ a \neq 0 \end{array}$$

$$\text{c) } \frac{4x^2}{3y^3} \times \frac{9y^2}{8x^4}$$

$$= \frac{36x^2y^2}{24y^3x^4}$$

$$= \frac{36}{24yx^2}$$

$$= \frac{3}{2yx^2}$$

$$x \neq 0, y \neq 0$$

$$d) \frac{a^2 + 5a + 6}{a^2 - 6a + 5} \times \frac{a^2 + a - 30}{a^2 + 9a + 18}$$

$$= \frac{(a+3)(a+2)}{(a-5)(a-1)} \times \frac{(a-5)(a+6)}{(a+6)(a+3)}$$

$$= \frac{\cancel{(a+3)}\cancel{(a+2)}\cancel{(a-5)}\cancel{(a+6)}}{\cancel{(a-5)}\cancel{(a-1)}\cancel{(a+6)}\cancel{(a+3)}}$$

$$= \frac{a+2}{a-1} \quad \begin{array}{l} a \neq 5 \\ a \neq 1 \\ a \neq -6 \\ a \neq -3 \end{array}$$

$$e) \frac{x^2 - x - 12}{x^2 - 9} \times \frac{x^2 - 4x + 3}{x^2 - 4x}$$

$$= \frac{(x-4)(x+3)}{(x+3)(x-3)} \times \frac{(x-1)(x-3)}{x(x-4)}$$

$$= \frac{\cancel{(x-4)}\cancel{(x+3)}\cancel{(x-1)}\cancel{(x-3)}}{\cancel{(x+3)}\cancel{(x-3)}\cancel{(x)}\cancel{(x-4)}}$$

$$= \frac{x-1}{x} \quad \begin{array}{l} x \neq -3 \\ x \neq 3 \\ x \neq 4 \\ x \neq 0 \end{array}$$

Dividing:

- Factor all numerators and denominators
- Convert into a multiplication question

$$\text{Ex) } \frac{2}{7} \div \frac{14}{5} = \frac{2}{7} \times \frac{5}{14}$$

- Simplify by eliminating common factors
- State all restrictions

Ex) Simplify the following.

$$\text{a) } \frac{x^2 - x - 20}{x^2 - 6x} \div \frac{x^2 + 9x + 20}{x^2 - 12x + 36}$$

$$= \frac{(x-5)(x+4)}{x(x-6)} \div \frac{(x+5)(x+4)}{(x-6)(x-6)} \left. \vphantom{\frac{(x-5)(x+4)}{x(x-6)}} \right\} \text{flip}$$

↑
change

$$= \frac{(x-5)\cancel{(x+4)}\cancel{(x-6)}(x-6)}{x\cancel{(x-6)}(x+5)\cancel{(x+4)}}$$

$$\boxed{\begin{array}{ll} x \neq 0 & , \quad x \neq -4 \\ x \neq -5 & \quad x \neq 6 \end{array}}$$

$$= \frac{(x-5)(x-6)}{x(x+5)}$$

$$b) \frac{a^2 - 3a - 28}{a^2 - 14a + 45} \div \frac{a^2 - 4a - 21}{a^2 - 2a - 15}$$

$$\frac{(a-7)(a+4)}{(a-9)(a-5)} \div \frac{(a-7)(a+3)}{(a-5)(a+3)} \left. \vphantom{\frac{(a-7)(a+4)}{(a-9)(a-5)}} \right\} \text{Flip}$$

Change

$$\frac{\cancel{(a-7)}(\cancel{a+4})\cancel{(a-5)}\cancel{(a+3)}}{\cancel{(a-9)}\cancel{(a-5)}\cancel{(a-7)}\cancel{(a+3)}}$$

$$= \frac{a+4}{a-9} \quad , \quad \begin{array}{l} a \neq 5 \\ a \neq 7 \end{array} \quad \begin{array}{l} a \neq 9 \\ a \neq -3 \end{array}$$

$$c) \frac{\frac{(x^2-4)}{(x+3)}}{\frac{(2x-4)}{(x^2+2x-3)}} \left. \vphantom{\frac{(x^2-4)}{(x+3)}} \right\} \frac{x^2-4}{x+3} \div \frac{2x-4}{x^2+2x-3}$$

$$= \frac{(x+2)(x-2)}{x+3} \div \frac{2(x-2)}{(x+3)(x-1)} \left. \vphantom{\frac{(x+2)(x-2)}{x+3}} \right\} \text{Flip}$$

Change

$$= \frac{(x+2)\cancel{(x-2)}\cancel{(x+3)}(x-1)}{\cancel{(x+3)}\cancel{(2)}\cancel{(x-2)}}$$

$$= \frac{(x+2)(x-1)}{2}$$

↑

$$\begin{array}{l} x \neq 3 \\ x \neq 2 \\ x \neq 1 \end{array}$$

$$d) \frac{2m^2 - 7m - 15}{2m^2 - 10m} \div \frac{4m^2 - 9}{6} \times (3 - 2m)$$

$$= \frac{(2m+3)(m-5)}{2m(m-5)} \overset{\ominus}{\div} \frac{(2m-3)(2m+3)}{6} \times (3-2m)$$

Change & flip

$$= \frac{\cancel{(2m+3)} \cancel{(m-5)} (6)(3-2m)}{(2m) \cancel{(m-5)} (2m-3) \cancel{(2m+3)}}$$

$$= \frac{(6)(3-2m)}{2m(2m-3)}$$

Very similar

$$= \frac{-1(6)(\cancel{2m-3})}{2m(\cancel{2m-3})}$$

$$\longrightarrow \frac{-6}{2m}$$

$$= -\frac{3}{m}$$

$$2m-3 \neq 0$$

$$\frac{2m}{2} \neq \frac{3}{2}$$

$$\sqrt{2m+3 \neq 0}$$

$$m \neq -\frac{3}{2}$$

Now Try

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11, 14, 15, 16

$$m \neq 1$$

$$m \neq 5$$

$$m \neq \frac{3}{2}, m \neq -\frac{3}{2}$$

Adding and Subtracting Rational Expressions:

When adding or subtracting fractions we must have a common denominator.

Ex)

$$\begin{aligned} \frac{1}{2} + \frac{3}{10} &= \frac{5}{10} + \frac{3}{10} && * 10 \text{ is the lowest common} \\ & && \text{multiple of 2 \& 10.} \\ &= \frac{8}{10} && \\ &= \frac{4}{5} && \leftarrow \text{reduce} \end{aligned}$$

When finding the lowest common multiple (or lowest common denominator) we must consider the coefficients and the variables involved.

Ex) Find the lowest common multiple of the following.

a) 2, 6

2 : 2, 4, 6, 8, ...

6 : 6, 12, 18

LCM: 6

c) a^2, a^5

LCM: a^5

b) 5, 3

LCM: 15

d) $8t^2, 12t^7$

$24t^7$

Ex) Simplify the following.

$$\text{a) } \frac{2}{3y} + \frac{4}{3y}$$

$$= \frac{6}{3y} \text{ reduce}$$

$$= \frac{2}{y}, y \neq 0$$

$$\text{b) } \frac{4t^2}{3} - \frac{6t^2}{3} + \frac{1}{3}$$

$$= \frac{-2t^2 + 1}{3}$$

$$\text{c) } \frac{2a}{3} + \frac{5a}{4} \text{ LCD: } 12$$

$$= \frac{8a}{12} + \frac{15a}{12}$$

$$= \frac{23a}{12}$$

$$\text{d) } \frac{y-5}{9} - \frac{y-2}{6} \text{ LCD: } 18$$

$$= \frac{2(y-5)}{18} - \frac{3(y-2)}{18}$$

$$= \frac{2y-10}{18} - \frac{3y-6}{18} \text{ Combine}$$

$$= \frac{-y-4}{18} = \frac{-(y+4)}{18}$$

$$\text{e) } \frac{5}{2a} + \frac{2a}{3} \text{ LCD: } 6a$$

$$\frac{15}{6a} + \frac{4a^2}{6a}$$

$$= \frac{15 + 4a^2}{6a}, a \neq 0$$

$$\text{f) } \frac{3x}{8x^4} - \frac{5}{6x^2} \text{ LCD: } 24x^4$$

$$\frac{9x}{24x^4} - \frac{20x^2}{24x^4}$$

$$= \frac{9x - 20x^2}{24x^4}$$

$$= \frac{x(9 - 20x)}{24x^4} = \frac{(9 - 20x)}{24x^3}, x \neq 0$$

$$g) \frac{6x+1}{5} + \frac{1}{1} - \frac{2x+4}{7} \quad \text{LCD: } 35$$

$$= \frac{7(6x+1)}{35} + \frac{35}{35} - \frac{5(2x+4)}{35}$$

$$= \frac{42x+7}{35} + \frac{35}{35} - \frac{10x+20}{35} \quad \text{Combine}$$

$$= \frac{32x+22}{35}$$

$$= \frac{2(16x+11)}{35}$$

Remember when adding or subtracting rational expressions we must have a common denominator.

To find the lowest common denominator we must first factor each denominator.

$$\frac{x}{2x-4} + \frac{3}{3x-6} = \frac{x}{2(x-2)} + \frac{3}{3(x-2)} \quad \text{LCD: } 6(x-2)$$

$$= \frac{3x}{6(x-2)} + \frac{6}{6(x-2)} \quad \text{Combine}$$

$$= \frac{3x+6}{6(x-2)} \quad \text{reduce}$$

$$= \frac{3(x+2)}{6(x-2)} = \frac{x+2}{2(x-2)} \quad x \neq 2$$

Ex) Determine the lowest common denominator for each of the following.

a) $\frac{1}{xy}, \frac{2}{3x^2y}$
 monomials - can't factor

LCD: $3x^2y$

b) $\frac{5x}{2x^2 - 6x}, \frac{3}{5x + 20}, \frac{4x}{x^2 + x - 12}$

→ can't tell if we have common factors

$\frac{5x}{2x(x-3)}, \frac{3}{5(x+4)}, \frac{4x}{(x+4)(x-3)}$

LCD: $10(x)(x-3)(x+4)$

Ex) Simplify the following.

a) $\frac{x+12}{4x-4} + \frac{2+x}{x-1}$ Factor denom.

$= \frac{x+12}{4(x-1)} + \frac{2+x}{x-1}$ LCD: $4(x-1)$

$= \frac{x+12}{4(x-1)} + \frac{4(2+x)}{4(x-1)}$ Equivalent fractions

$= \frac{x+12}{4(x-1)} + \frac{8+4x}{4(x-1)}$ Combine

$= \frac{x+12+8+4x}{4(x-1)}$

$= \frac{5x+20}{4(x-1)}$ Reduce, if possible → $= \frac{5(x+4)}{4(x-1)}$ Restrict $x \neq 1$

$$\text{c) } \frac{4}{y^2 + 5y + 6} - \frac{5}{y^2 - y - 12}$$

$$\text{d) } \frac{x+5}{x^2 - 3x - 10} - \frac{x-1}{x^2 - 9x + 20}$$

$$e) \frac{y^2 - 20}{y^2 - 4} + \frac{y - 2}{y + 2}$$

$$f) \frac{1 + \frac{1}{x}}{x - \frac{1}{x}}$$

$$\left(1 + \frac{1}{x}\right) \div \left(x - \frac{1}{x}\right)$$

$$\left(\frac{x}{x} + \frac{1}{x}\right) \div \left(\frac{x^2}{x} - \frac{1}{x}\right)$$

$$\left(\frac{x+1}{x}\right) \div \left(\frac{x^2-1}{x}\right)$$

$$\left(\frac{x+1}{x}\right) \cdot \left(\frac{x}{x^2-1}\right)$$

$$\frac{x+1}{x^2-1}$$

$$\frac{\cancel{x+1}}{(x+1)(x-1)} \cdot \frac{1}{x-1}$$

$x \neq 1$
 $x \neq 0$
 $x \neq -1$

$$g) \frac{\frac{5}{x^2-16} + \frac{2x}{x^2+2x-24}}{\frac{7x}{x^2+10x+24} - \frac{3}{x-4}}$$

$$\left[\frac{5}{x^2-16} + \frac{2x}{x^2+2x-24} \right] \div \left[\frac{7x}{x^2+10x+24} - \frac{3}{x-4} \right]$$

$$\left[\frac{5}{(x+4)(x-4)} + \frac{2x}{(x+6)(x-4)} \right] \div \left[\frac{7x}{(x+4)(x+6)} - \frac{3}{x-4} \right]$$

$$\frac{5(x+6) + 2x(x+4)}{(x+4)(x-4)(x+6)} \div \frac{7x(x-4) - 3(x+4)(x+6)}{(x+4)(x-4)(x+6)}$$

$$\frac{5x+30+2x^2+8x}{(x+4)(x-4)(x+6)} \div \frac{7x^2-28x-3x^2-30x-72}{(x+4)(x-4)(x+6)}$$

$$\frac{2x^2+13x+30}{(x+4)(x-4)(x+6)} \div \frac{4x^2-58x-72}{(x+4)(x-4)(x+6)}$$

$$\frac{2x^2+13x+30}{(x+4)(x-4)(x+6)} = \frac{\cancel{(x+4)}(x-4)(x+6)}{4x^2-58x-72}$$

$$= \frac{2x^2+13x+30}{4x^2-58x-72} \leftarrow \text{can't factor}$$

$$= \frac{2x^2+13x+30}{2(2x^2-29x-36)}$$

$$x \neq -1, 15, \dots$$

$$x \neq 4$$

$$x \neq -6$$

Now Try

Page 336 #1, 4, 5, 6, 7,

8, 10, 12, 15

Solving Rational Equations:

- Factor all denominators
- Determine the lowest common denominator
- Multiply both sides of the equation by the lowest common denominator (this will eliminate all fractions)
- Solve as normal
- Check for extraneous roots (your answer cannot be a restriction)

Ex) Solve the following.

a) $\frac{x}{2} + 3 = 2 + \frac{3x}{4}$

b) $\frac{t+5}{8} = t + \frac{3}{2}$

c) $\frac{x-1}{2x} + \frac{1}{x} = 2$

d) $\frac{4}{2x-1} = \frac{1}{x-2}$

$$\text{e) } \frac{1}{m-2} = \frac{5}{m+4}$$

$$\text{f) } \frac{4}{x} + 6 = 2$$

$$\text{g) } \frac{5}{2y} + \frac{11}{12} = \frac{2}{3y}$$

$$\text{h) } \frac{2}{x^2-4} + \frac{10}{6x+12} = \frac{1}{x-2}$$

$$\text{i) } \frac{9}{y-3} - \frac{4}{y-6} = \frac{18}{y^2-9y+18}$$

$$\text{j) } \frac{3}{x^2 - x - 2} + \frac{4}{x^2 - 4} = \frac{1}{x^2 + 3x + 2}$$

$$\text{k) } \frac{4k - 1}{k + 2} - \frac{k + 1}{k - 2} = \frac{k^2 - 4k + 24}{k^2 - 4}$$

Now Try

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Word Problems Involving Rational Equations:

- Identify the variable being used. This can be done with a let statement, a table, or a diagram
- Create an equation that describes the situation
- Solve for the variable
- Check to see that you have answered the question and that your answer makes sense
- Answer the question with a sentence

Ex) Find two consecutive numbers where half of the smaller is equal to 4 more than one third the larger. \mathcal{B}

$$1^{\text{st}} \# : x$$

$$2^{\text{nd}} \# : x+1$$

$$\frac{x}{2} = \frac{x+1}{3} + 4$$

$$\frac{6x}{2} = \frac{6(x+1)}{3} + 24$$

$$3x = 2(x+1) + 24$$

$$3x = 2x + 2 + 24$$

$$x = 26$$

$$2^{\text{nd}} = 27$$

Ex) The average life span of a woodland caribou is 5 years + longer than half the average life span of a moose. The sum of their life spans is 35 years. What is the life span of a moose?

$$m : x$$

$$c : \frac{x}{2} + 5$$

$$x + \frac{x}{2} + 5 = 35$$

Ex) Two friends share a paper route. Sheena can deliver the papers in 40 min. Jeff can cover the same route in 50 min. How long, to the nearest minute, does the paper route take if they work together?

return → Ex) In a particular dog race from Pas to Flin Flon the total distance covered was 140 miles. Conditions were excellent on the way to Flin Flon. However, bad weather caused the winner's average speed to decrease by 6 mph on the return trip. The total time for the trip was $8\frac{1}{2}$ hours. What was the winning dog team's average speed on the way to Flin Flon? $s = \frac{d}{t} \leftarrow t = \frac{d}{s}$

Time there:

$$t = \frac{70}{x}$$

$$\frac{70}{x} + \frac{70}{x-6} = 8.5$$

Time back:

$$t = \frac{70}{x-6}$$

Now Try

Page 348 #8, 9, 10, 12,
13, 14, 15, 18,
19, 21, 27

Reciprocal Functions:

A reciprocal of a value is the “flip” of the fraction:

$$\text{Ex) } \frac{2}{3} \xrightarrow{\text{reciprocal}} \frac{3}{2}$$

$$5 \xrightarrow{\text{reciprocal}} \frac{1}{5}$$

$$\frac{-1}{x} \xrightarrow{\text{reciprocal}} \frac{x}{-1} = \underset{\uparrow}{-x}$$

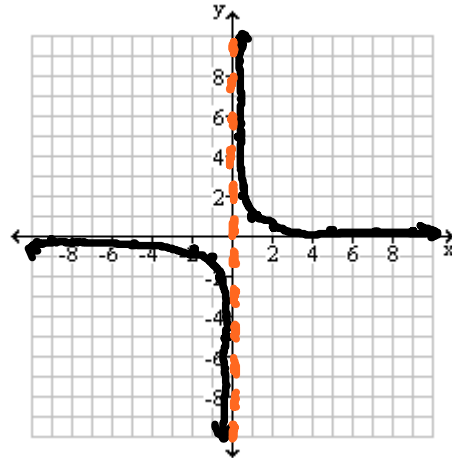
A reciprocal of a function is found by dividing ‘1’ by the function.

If $y = f(x)$, then $y = \frac{1}{f(x)}$ is its reciprocal.

$$y = x - 3 \rightarrow y = \frac{1}{x - 3}$$

Ex) Complete the table of values given below, then use this to sketch the graphs of $y = x$ and $y = \frac{1}{x}$.

| x | $y = x$ | $y = \frac{1}{x}$ |
|----------------|----------------|----------------------------------|
| -10 | -10 | $-\frac{1}{10}$ |
| -5 | -5 | $-\frac{1}{5}$ |
| -2 | -2 | $-\frac{1}{2}$ |
| -1 | -1 | $-\frac{1}{1} = -1$ |
| $-\frac{1}{2}$ | $-\frac{1}{2}$ | -2 |
| $-\frac{1}{5}$ | $-\frac{1}{5}$ | -5 |
| 0 | 0 | $\frac{1}{0} = \text{undefined}$ |
| $\frac{1}{5}$ | $\frac{1}{5}$ | 5 |
| $\frac{1}{2}$ | $\frac{1}{2}$ | 2 |
| 1 | 1 | 1 |
| 2 | 2 | $\frac{1}{2}$ |
| 5 | 5 | $\frac{1}{5}$ |
| 10 | 10 | $\frac{1}{10}$ |



Handwritten notes and scribbles on the left side of the page. There are several 'm' characters written vertically. Below them, there are some scribbles in red, orange, and blue. A line points from the word 'Characteristics' to the table.

Characteristics

Non-Permissible Values *these are restrictions*

Asymptotes *imaginary lines that your graph approaches but doesn't touch*

Invariant Points

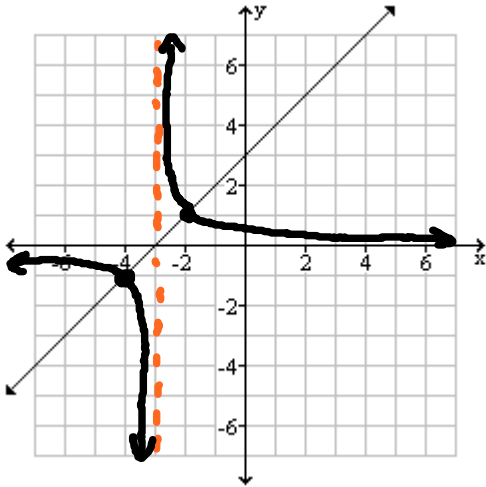
Points that appear on $y = x$ & $y = \frac{1}{x}$

Properties / Rules:

| $y = f(x)$ (original) | $y = \frac{1}{f(x)}$ (reciprocal) |
|----------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|
| If $f(x) = 0$ | then $\frac{1}{f(x)}$ is $\frac{1}{0} = \text{undefined}$ and a <u>vertical asymptote</u> may exist. |
| If $f(x)$ is undefined and has a vertical asymptote. | then $\frac{1}{f(x)} = \frac{0}{1} = 0$. $f(x) = \frac{1}{0}$ |
| If $f(x) = 1$ | then $\frac{1}{f(x)} = \frac{1}{1} = 1$. |
| If $f(x) = -1$ | then $\frac{1}{f(x)} = \frac{1}{-1} = -1$. |
| If $f(x)$ is positive | then $\frac{1}{f(x)}$ is <u>+</u> . $+f(x)$ |
| If $f(x)$ is negative | then $\frac{1}{f(x)}$ is <u>-</u> . |
| If $f(x)$ is increasing | then $\frac{1}{f(x)}$ is <u>decreasing</u> . |
| If $f(x)$ is decreasing | then $\frac{1}{f(x)}$ is <u>increasing</u> . |
| If $f(x)$ approaches 0 | then $\frac{1}{f(x)}$ approaches ∞ . |
| If $f(x)$ approaches $\pm\infty$ (∞ means infinity) | Then $\frac{1}{f(x)}$ approaches 0 . |

Ex) Given the graph of $y = f(x) = x + 3$, sketch the

graph of $y = \frac{1}{f(x)} = \frac{1}{x+3}$



Steps: ① Draw asymptotes anywhere graph crosses x-axis ($y=0 \therefore \frac{1}{0} = \text{undefined}$)

② Label invariant points (where would $f(x) = \frac{1}{f(x)}$?)

$$f(x) = 1, \frac{1}{f(x)} = \frac{1}{1} = 1$$

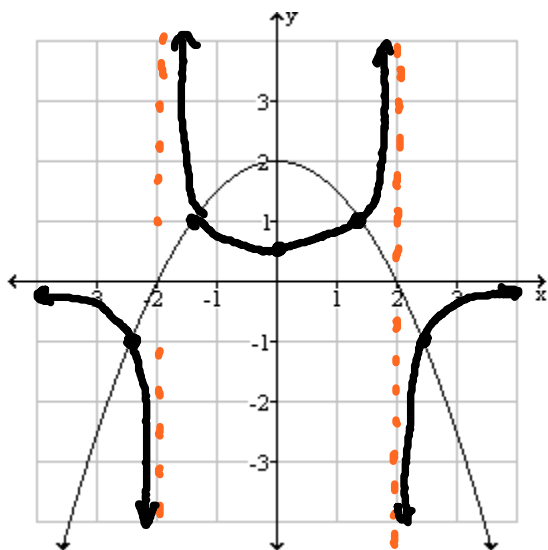
③ Determine $\frac{1}{f(x)}$ for any vertex/sharp edge.

④ Draw:
 \rightarrow As $f(x) \rightarrow \infty, \frac{1}{f(x)} \rightarrow 0$
 \rightarrow As $f(x) \rightarrow -\infty, \frac{1}{f(x)} \rightarrow 0$

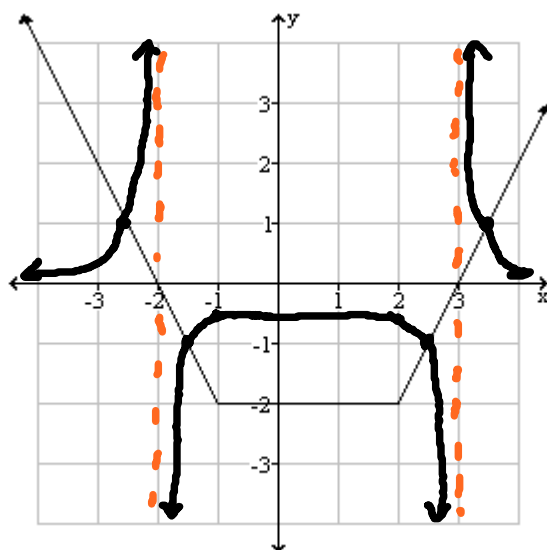
Ex) For each of the following, sketch the graph of

$$y = \frac{1}{f(x)} \text{ given the graph of } y = f(x).$$

a)



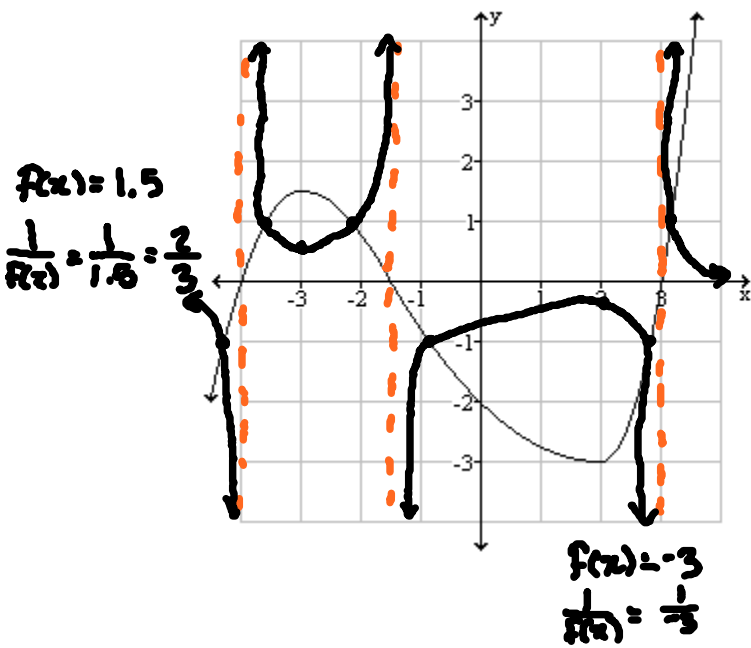
b)



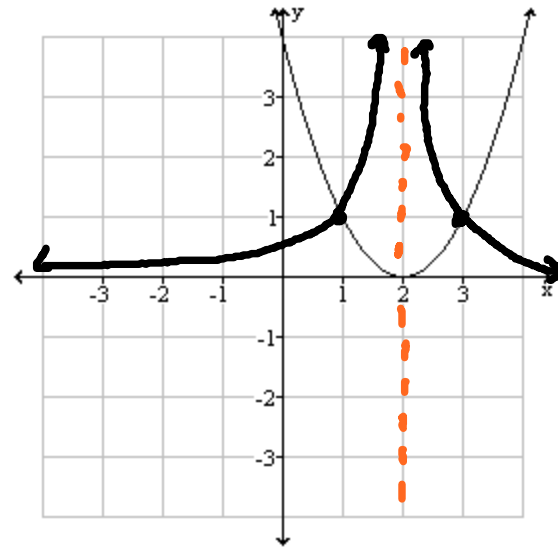
$$f(x) = -2 \therefore \frac{1}{f(x)} = \frac{1}{-2} = -\frac{1}{2}$$

Vertex
 $f(x) = 2$
 $\frac{1}{f(x)} = \frac{1}{2}$

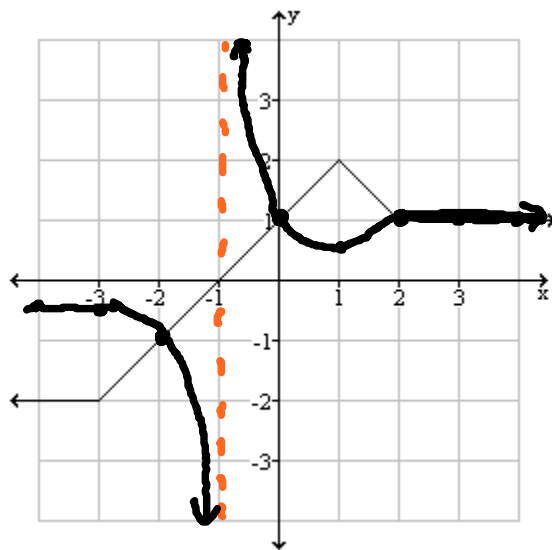
c)



d)



e)



$f(x) = 2$
 $\frac{1}{f(x)} = \frac{1}{2}$

$f(x) = -2$
 $\frac{1}{f(x)} = -\frac{1}{2}$

Now Try

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