Unit 3 Radical and Absolute Value Functions

Working with Radicals:

Radicals include square roots, cube roots, 4th roots, etc.

square root · index 2

$\sqrt{\ } \rightarrow square-root$

$\sqrt[3]{\ } \rightarrow cube-root$

$\sqrt[4]{\ } \rightarrow fourth-root$

$\sqrt[5]{\ } \rightarrow 5th-root$

$\sqrt[index]{radicand}$

index

$\sqrt[2]{9}$

radicand

Like Radicals:

Radicals with the same radicand and index are called like radicals.

Same index
Same radicand

Ex) Like Radicals

$5\sqrt{7}$ and $-\sqrt{7}$

$\frac{2}{3}\sqrt[3]{5x^2}$ and $\sqrt[3]{5x^2}$

Unlike Radicals

$2\sqrt{5}$ and $2\sqrt{3}$

$\sqrt[4]{5a}$ and $\sqrt[3]{5a}$

Like radicals can be combined together the same way like terms are combined together.

Ex) $3\sqrt{5}-12\sqrt{5}+\sqrt{7}+\sqrt{5}+4\sqrt{7}$

Reducing Radicals:

When we simplify radicals we make an entire radical into a mixed radical.

Ex) Simplify the following radicals.

$\sqrt{60}$ $\sqrt{180}$ $\sqrt{162}$ $\sqrt[3]{120}$

$\sqrt{48y^5}$ $\sqrt[4]{c^9}$ $\sqrt[3]{320x^4y^2}$

Creating Entire Radical:

When going from a mixed radical to an entire radical you just do the opposite.

Ex) Express the following as an entire radical

$3\sqrt{15}$ $5\sqrt{10}$ $6\sqrt[3]{12}$ $5b\sqrt[3]{ab^2}$

Multiplying Radicals:

To multiply radicals with the same index together, simply multiply the coefficients together then multiply the radicands together. (Your final answer should be in reduced form.)

Ex) Multiply the following.

$9\sqrt{2}\times 4\sqrt{7}$ $\qquad$ $2\sqrt{3}\times 5\sqrt{6}$ $\qquad$ $2\sqrt{5x}\times 3\sqrt{3}\times\sqrt{6x^2}$

Ex) Simplify the following radicals.

$\sqrt{50}+3\sqrt{2}$ $\qquad$ $-\sqrt{27}+3\sqrt{5}-\sqrt{80}-2\sqrt{12}$

$\sqrt{4c}-4\sqrt{9c}$

Now Try
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Multiplying Radicals (Continued):

Remember to reduce radicals when possible after multiplying.

Ex) Simplify the following.

a) $7\sqrt{3}\left(5\sqrt{5}-6\sqrt{3}\right)$

$(7\sqrt{3})(5\sqrt{5})-(7\sqrt{3})(6\sqrt{3})$

$35\sqrt{15}-42\sqrt{9}$

$35\sqrt{3\times5}-42\sqrt{3\times3}$

$35\sqrt{15}-42(3) \rightarrow \boxed{35\sqrt{15}-126}$

b) $\left(8\sqrt{2}-5\right)\left(9\sqrt{5}+6\sqrt{10}\right)$

$=(8\sqrt{2})(9\sqrt{5})+(8\sqrt{2})(6\sqrt{10})-(5)(9\sqrt{5})-(5)(6\sqrt{10})$

$=72\sqrt{10}+48\sqrt{20}-45\sqrt{5}-30\sqrt{10}$

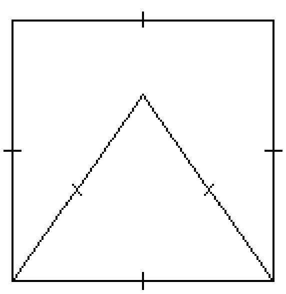
$=72\sqrt{10}+48\sqrt{2\times2\times5}-45\sqrt{5}-30\sqrt{10}$

$=72\sqrt{10}+96\sqrt{5}-45\sqrt{5}-30\sqrt{10}$

$\boxed{=42\sqrt{10}+51\sqrt{5}}$

c) $9\sqrt[3]{2w}\left(\sqrt[3]{4w}+7\sqrt[3]{28}\right)$

Ex) The area of the square shown below is 32 cm^2.



a) What is the exact perimeter of the triangle?

b) Determine the height of the triangle.

c) What is the exact area of the triangle?

Dividing Radicals:

Division of radicals works the same as multiplication. To divide radicals with the same index together, simply divide the coefficients, then divide the radicands. (Your final answer should be in reduced form.)

Ex) Simplify the following.

$$\div \frac{36\sqrt{15}}{9\sqrt{3}} \div \qquad\qquad \frac{28\sqrt{150}}{2\sqrt{3}} \quad 70\sqrt{2}$$

$= 4\sqrt{5}$ Can you reduce?

Rationalizing Denominators Part I:

When dealing with fractions we do not leave radicals in the denominator. (Radicals in the numerator are OK.)

Ex) $\frac{7}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ We really just multiplied by 1!

$= \frac{7\sqrt{5}}{\sqrt{25}}$ ← This will always happen

$\boxed{= \frac{7\sqrt{5}}{5}}$

Ex) Rationalize the following.

a) $\dfrac{4\sqrt{3}}{\sqrt{7}} \times \dfrac{\sqrt{7}}{\sqrt{7}}$

$\dfrac{4\sqrt{21}}{7}$

b) $\dfrac{5}{3\sqrt{11}}$

c) $\dfrac{9\sqrt{24}}{\sqrt{6x}}$

d) $\dfrac{5}{8\sqrt[3]{x}} \times \dfrac{\sqrt[3]{x}}{\sqrt[3]{x}} \times \dfrac{\sqrt[3]{x}}{\sqrt[3]{x}}$

$= \dfrac{5\sqrt[3]{x}}{8\sqrt[3]{x^2}} \times \dfrac{\sqrt[3]{x}}{\sqrt[3]{x}}$

$= \dfrac{5\sqrt[3]{x^2}}{8\sqrt[3]{x^3}}$

$\boxed{= \dfrac{5\sqrt[3]{x^2}}{8x}}$

Rationalizing Denominators Part II:

When two terms appear in the denominator that involve a radical a different approach is needed.

Simplify the following.

$\left(3-4\sqrt{5}\right)\left(3+4\sqrt{5}\right)$

$= (3)(3) + \cancel{(3)(4\sqrt{5})} - \cancel{(4\sqrt{5})(3)} - (4\sqrt{5})(4\sqrt{5})$

$= 9 - 16\sqrt{25}$

$= 9 - 80$

$= -71$

Sign is opposite to start. It is called multiplying by the conjugate.

$\left(\sqrt{7}+2\sqrt{3}\right)\left(\sqrt{7}-2\sqrt{3}\right)$

$= \sqrt{49} - 4\sqrt{9}$

$= 7 - 4(3)$

$= 7 - 12$

$= -5$

We have conjugate So we know middle terms will cancel

Ex) $\frac{5}{6+\sqrt{7}} \times \frac{6-\sqrt{7}}{6-\sqrt{7}}$ Mult by conjugate

$= \frac{30-5\sqrt{7}}{36-\sqrt{49}}$

$= \frac{30-5\sqrt{7}}{36-7}$

$\frac{30-5\sqrt{7}}{29}$

Ex) Rationalize the following denominators

a) $\frac{5\sqrt{3}}{4+\sqrt{6}} \times \frac{4-\sqrt{6}}{4-\sqrt{6}}$

b) $\frac{11}{\sqrt{5}-7} \times \frac{\sqrt{5}+7}{\sqrt{5}+7}$

c) $\frac{\sqrt{3}-6}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

d) $\frac{3\sqrt{10}}{4\sqrt{5}-2} \times \frac{4\sqrt{5}+2}{4\sqrt{5}+2}$

Now Try
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11, 13, 17, 19,

Solving Radical Equations:

A radical equation is one in which a variable appears inside a radical.

Ex) $\sqrt{x}+3=15$, $8-\sqrt{x+4}=x+7$

Equations involving 1 Radical:

Steps for Solving:

- Isolate (solve for) the radical.
- Square both sides of the equation.
- Solve.
- Check for Extraneous Roots.

Ex) Solve the following radical equations. Check for extraneous roots.

a) $\sqrt{x+1}+3=5$

$\sqrt{x+1}=2$

$\cancel{(\sqrt{x+1})^2}=2^2$

$x+1=4$

$\boxed{x=3}$

Extraneous?

$\sqrt{3+1}+3=5$

$\sqrt{4}+3=5$

$2+3=5$

$5=5$ ✓

no extraneous Solutions.

b) $x=\sqrt{x-2}+4$

$x-4=\sqrt{x-2}$

$(x-4)^2=\cancel{(\sqrt{x-2})^2}$

$(x-4)(x-4)=x-2$

$x^2-8x+16=x-2$

$x^2-9x+18=0$ ← Factor, graph, quad

$(x-6)(x-3)=0$

$x=6$ $\cancel{x=3}$ Extraneous

Check

Equations involving 2 Radicals:

Steps for Solving:

- Isolate (solve for) one of the radicals.
- Square both sides of the equation.
- Isolate (solve for) the other radical.
- Square both sides of the equation.
- Solve.
- Check for Extraneous Roots.

Ex) Solve the following radical equations. Check for extraneous roots.

a) $\sqrt{4x+5}-\sqrt{2x-1}=2$

b) $3+\sqrt{x-4}=\sqrt{x+11}$

$(3+\sqrt{x-4})(3+\sqrt{x-4}) = (\cancel{\sqrt{x+11}})^{\cancel{2}}$

$9+3\sqrt{x-4}+3\sqrt{x-4}+x-4 = x+11$

$6\sqrt{x-4}+\cancel{x}+5 = \cancel{x}+11$

$\frac{6\sqrt{x-4}}{6} = \frac{6}{6}$

$\sqrt{x-4} = 1$

$(\cancel{\sqrt{x-4}})^{\cancel{2}} = 1^2$

$x-4 = 1$

$x = 5$

Check

$3+\sqrt{5-4} = \sqrt{5+11}$

$3+\sqrt{1} = \sqrt{16}$

$3+1 = 4$

$4 = 4$ ✓

c) $\sqrt{x-1}-\sqrt{5-x}=0$

$\sqrt{x-1} = \sqrt{5-x}$

$(\cancel{\sqrt{x-1}})^{\cancel{2}} = (\cancel{\sqrt{5-x}})^{\cancel{2}}$

$x-1 = 5-x$

$\frac{2x}{2} = \frac{6}{2}$

$\boxed{x=3}$

Check

$\sqrt{3-1}-\sqrt{5-3} = 0$

$\sqrt{2}-\sqrt{2} = 0$

$0 = 0$ ✓

Now Try
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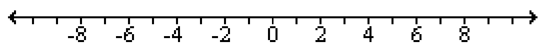
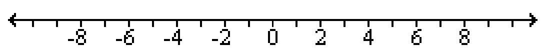
Absolute Value:

The absolute value of a number can be thought of as the distance from that number to zero on a number line. The absolute value is a positive number.

Ex) a) Determine the absolute value of -6



b) Determine the absolute value of 5



To symbolize absolute value when use vertical bars around a number or expression

$|-14|$ means the absolute value of -14

In general the absolute value of a real number *x* is defined as

$$\boxed{|x| = \begin{cases} x\ , & x \geq 0 \\ -x\ , & x < 0 \end{cases}}$$

Ex) Evaluate the following.

a) $|3|$ b) $|-11|$ c) $|28-6|$

d) $|4|-|-6|$ e) $5-3|2-7|$ f) $|-2(5-7)^2+6|$

Ex) On stock markets, individual stock and bond values fluctuate a great deal. A particular stock opened the month at $13.55 per share, dropped to $12.70, increased to $14.05, and closed the month at $13.85. Determine the total change in the value of this stock for the month. This total shows how active the stock was that month.

Now Try
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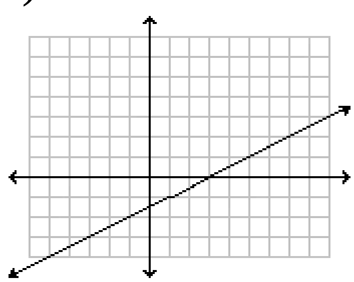
Absolute Value Functions and Their Graphs:

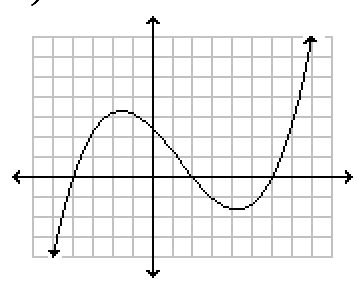
- When graphing an absolute value function we can begin my considering the graph of the function inside the absolute value .
- To obtain the graph of the absolute value function simply flip up any sections of the graph that are below the *x*-axis. (Absolute value results in a positive value.)

Ex) Given the graph of $y = f(x)$ sketch the graph of $y = |f(x)|$

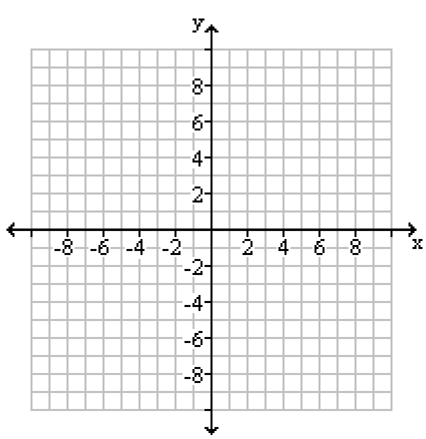
a)



b)



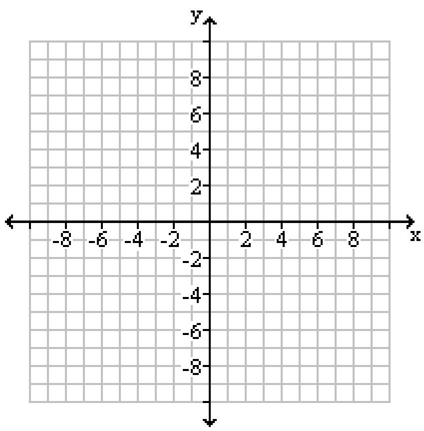
Ex) Sketch the graph of $y = |2x - 3|$, then write $y = |2x - 3|$ as a piecewise function.



Ex) Consider the absolute value function $y = \left|-x^2 + 2x + 8\right|$

a) Determine the y-intercept and the x-intercept

b) Sketch the graph



b) State the domain and range.

c) Express as a piecewise function.

Now Try
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Solving Absolute Value Equations:

Absolute value equations are equations that contain a variable inside an absolute value sign.

Ex) $|x|-2=0$, $3|x-7|=x-5$

Steps for Solving:

- Break the equation up into 2 equations. In one case consider what is in the absolute value sign to be positive, and in the other consider it to be negative.
- Solve each equation.
- Check each solution for Extraneous Roots

Ex) Solve the absolute value equations given below. Check for extraneous roots.

a) $|2x-3|=5$

b) $|x-4|=2x+1$

c) $2x-3=|x-3|$

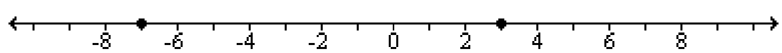
d) $|2x-5|=5-3x$

d) $|3x-4|+12=9$

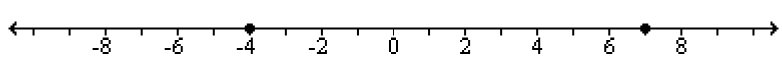
Ex) Kokanee salmon are sensitive to water temperature. If the water is too cold, egg hatching is delayed, and if the water is too warm, the eggs die. Biologists have found that the spawning rate of the salmon is greatest when the water is at an average temperature of 11.5°C with an absolute value difference of 2.5°C. Write and solve the limits of the ideal temperature range for the Kokanee salmon to spawn.

Ex) Determine an absolute value equation in the form $|ax+b|=c$ given its solutions on the number line.

a)



b)



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