Unit 1 Quadratic Functions & Equations

<u>Graphing Quadratics Part I:</u>

What is a Quadratic?

A quadratic is an expression of degree 2. (exp)

Ex) $y = x^2 + 3$ $\qquad$ $y = (x+3)(x-7)$

$\qquad$ $y = (x+4)^2 - 7$ $\qquad$ $= x^2 - 7x + 3x - 21$

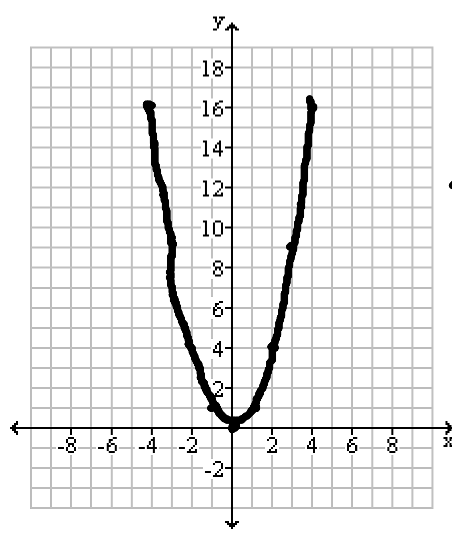
Graph of $y = x^2$:

$\boxed{y=}$ $\boxed{2^{nd}}$ $\boxed{\text{Graph}}$

The most basic quadratic function is given by $y = x^2$.

Create a table of values for $y = x^2$ and then graph it on the grid provided.

x	y
–4	$(-4)(-4) = 16$
–3	$(-3)(-3) = 9$
–2	4
–1	1
0	0
1	1
2	4
3	9
4	16



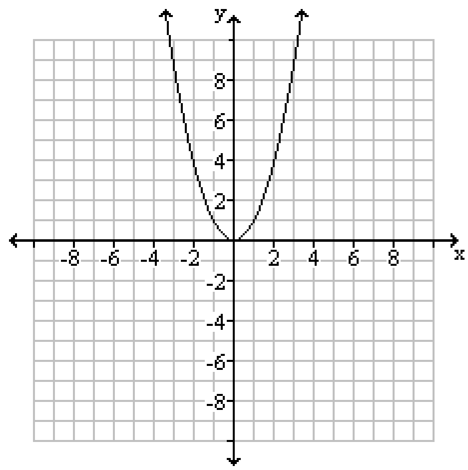
Parabola

↳ Symmetry

Graphing Quadratics of the form $y = x^2 + q$:

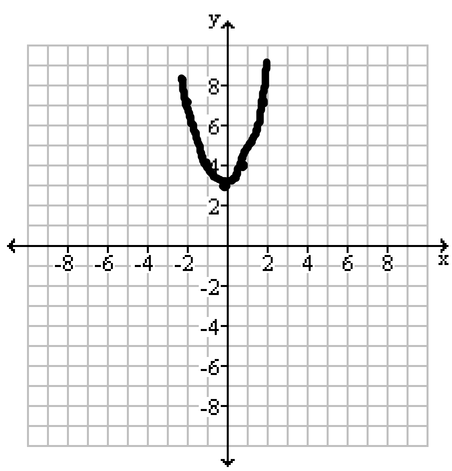
constant

Below is the graph of $y = x^2$

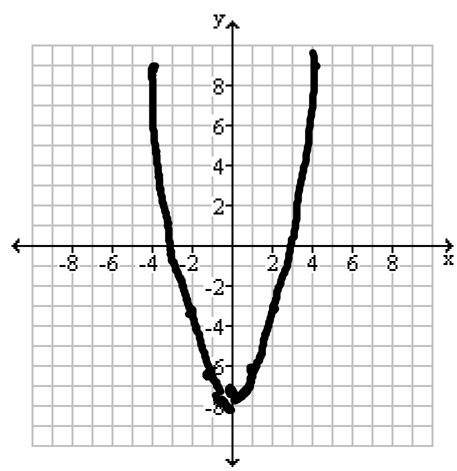


Original

Sketch the graph of
$y = x^2 + 3$



Sketch the graph of
$y = x^2 - 7$



Rule: When we add a constant to x^2, our graph shifts up by that many. When we subtract a constant, the graph shifts down by that many.

Function Notation:

Our original graph $y=f(x)$

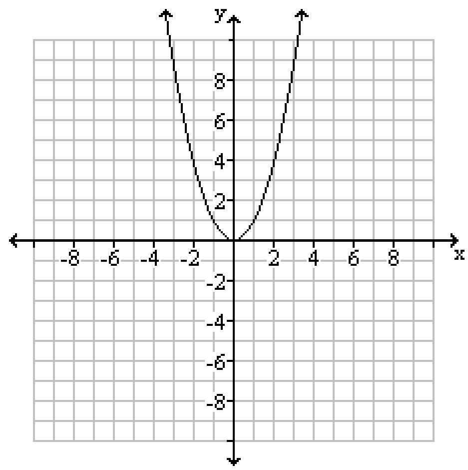
Our new graph: $y=f(x)+q$ *

when $q>0$, shifts up

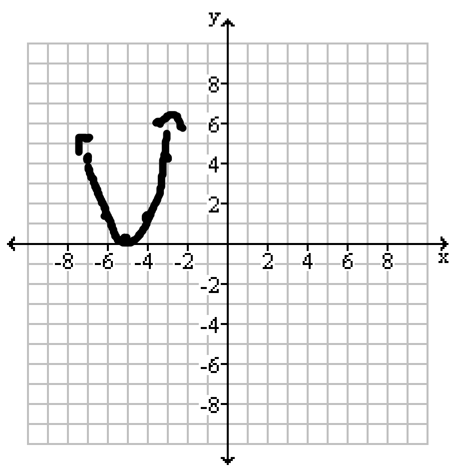
$q<0$, shifts down

Graphing Quadratics of the form $y = (x - p)^2$:

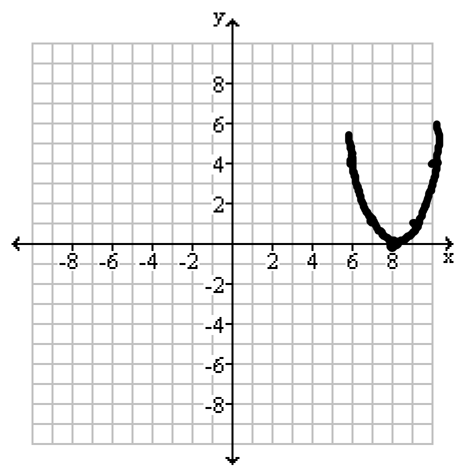
Below is the graph of $y = x^2$



Sketch the graph of $y = (x+5)^2$



Sketch the graph of $y = (x-8)^2$



Rule: When we add a constant directly to x, our graph shifts left. When we subtract a constant directly from x, our graph shifts right.

Function Notation:
Original: $y = f(x)$
Our new: $y = f(x+p)$
$p>0$, shift left
$p<0$, shift right.

In general if $\boxed{y=(x+p)^2+q}$ then

$p>0$ the graph moves left

$p<0$ the graph moves right

$q>0$ the graph moves up

$q<0$ the graph moves down

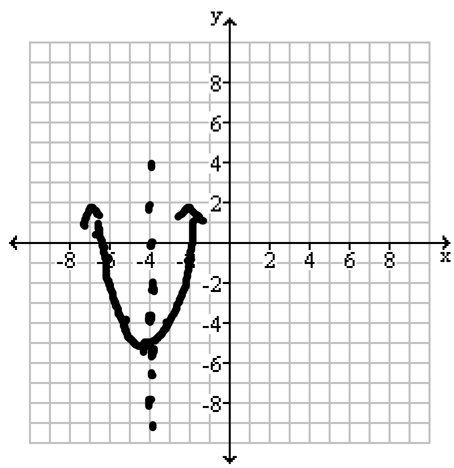
The vertex of the parabola is given by $(\underset{x}{-p}, \underset{y}{q})$.

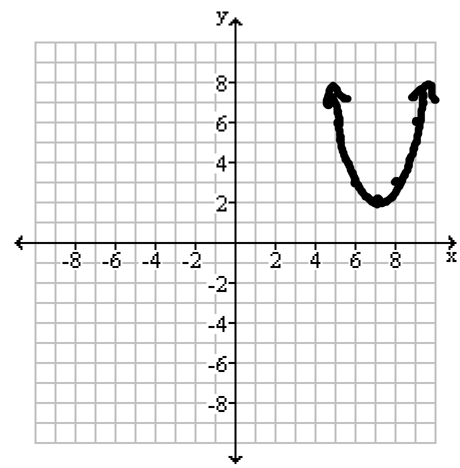
Ex) Sketch the following parabolas. Then answer the following.

a) $y=(x+4)^2-5$ — 4 left, 5 down

b) $y=(x-7)^2+2$ — 7 right, 2 up

Vertex $(-4,-5)$





Vertex: $(7,2)$

Coordinates of Vertex: $(-4,-5)$

Axis of Symmetry: $x=-4$

x Domain: $x \in \mathbb{R}$

y Range: $y \geq -5$

x-intercept(s): $(-6.23, 0)$, $(-1.76, 0)$

2nd / Trace / Zero

y-intercept: $x=0$ $(0,11)$

$y=(x+4)^2-5$

$=(4)^2-5$

Coordinates of Vertex: $(7,2)$

Axis of Symmetry: $x=7$

Domain: $x \in \mathbb{R}$

Range: $y \geq 2$

x-intercept(s): None

y-intercept: $x=0$

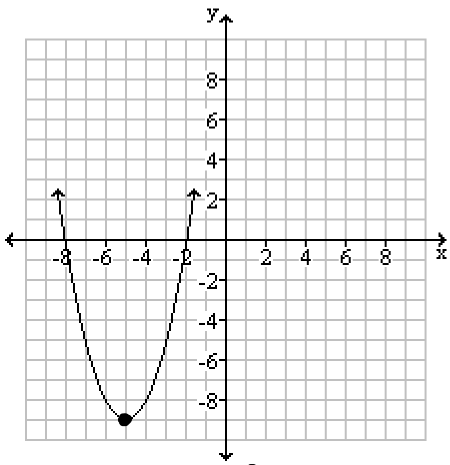
$y=(x-7)^2+2$

$=(-7)^2+2$

$=51$ $(0,51)$

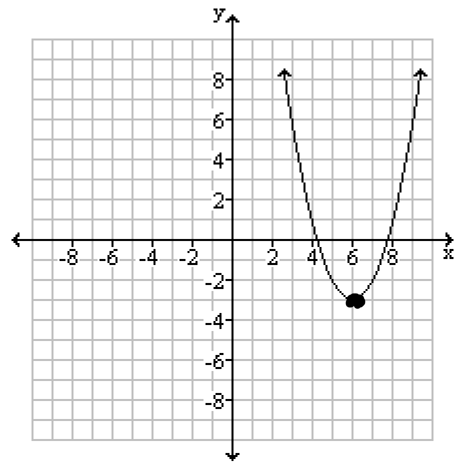
Ex) Determine the equation of the following parabolas.

a)



$y=(x+5)^2-9$

b)



$y=(x-6)^2-3$

Ex) Describe what happens to the point $(-4,\ 16)$ on the graph of $y=x^2$ when the graph of $y=x^2$ is changed into the graph of $y=(x+7)^2+18$.

→ 7 left

→ 18 up

$(-4, 16)$
x y

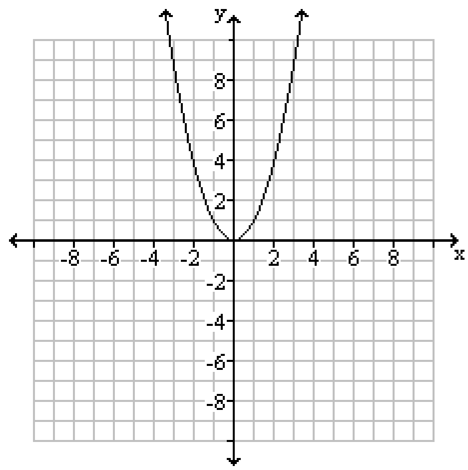
$\boxed{(-11, 34)}$

Now Try
Worksheet

Graphing Quadratics Part II:

Graphing Quadratics of the form $y = ax^2$:

Below is the graph of $y = x^2$



Sketch the graph of

$y = 3x^2$

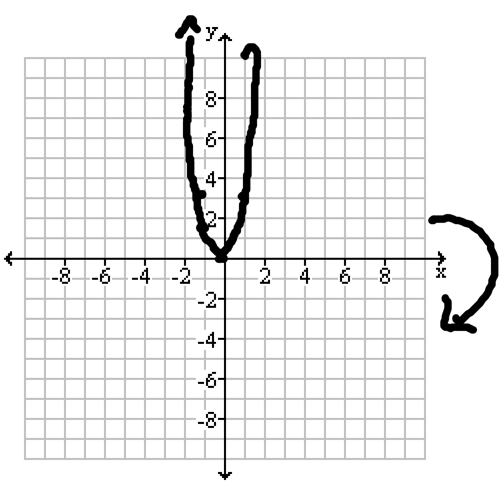
Old → New

(1,1) → (1, 3)

×3

(2,4) → (2, 12)

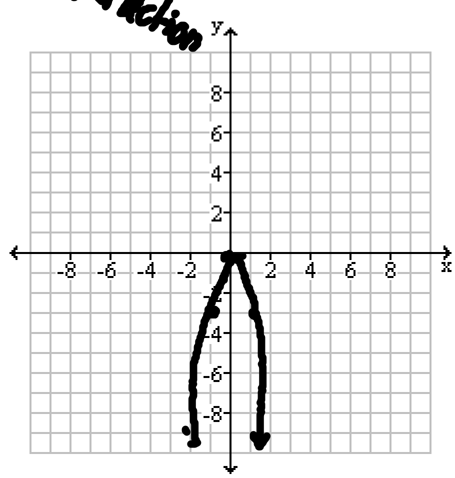
×3



Sketch the graph of

$y = \boxed{-}3x^2$

reflection

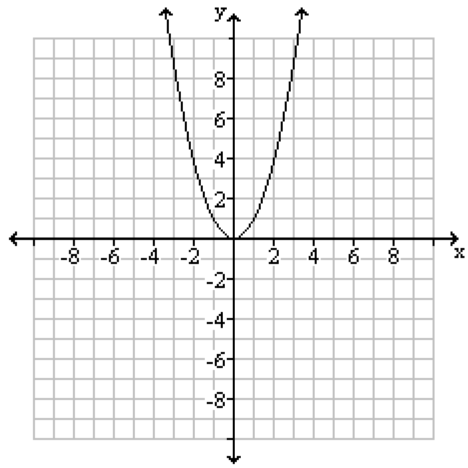


Old → New

(1,1) → (1, −3)

(2,4) → (2, −12)

Below is the graph of $y = x^2$



Sketch the graph of

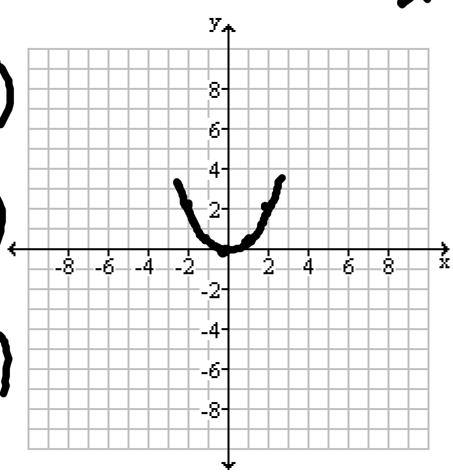
$y = \frac{1}{2}x^2$ $0.5x^2$

$(0,0) \to (0,0)$

$(1,1) \to (1,0.5)$

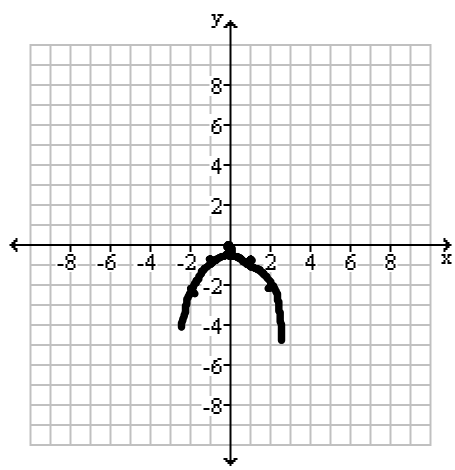
$(2,4) \to (2,2)$

$(3,9) \to (3,4.5)$



Sketch the graph of

$y = -\frac{1}{2}x^2$



Rule:

→ When we mult x^2 by a constant, our original y values get multiplied by that constant to give our new y value

→ when constant > 1, graph becomes steep.

→ " " < 1, graph compresses (flatter)

→ – in front means reflection over x-axis

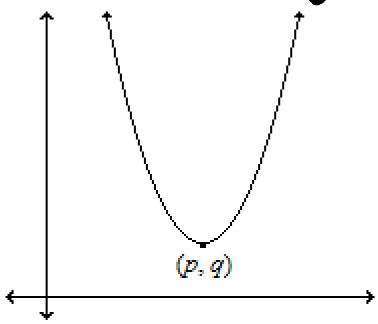
Function Notation:

Original : $y = f(x)$

New : $y = af(x)$ where a is a constant & a tells us $\boxed{\text{stretch factor}}$

In general if $y = a(x-p)^2 + q$ then

+ Stretch



$(p,\ q)$ gives Vertex

if $a > 0$ then open up

if $a < 0$ then opens down

a also determines Stretch factor

Ex) Without using your calculator, graph the following quadratics and determine its characteristics.

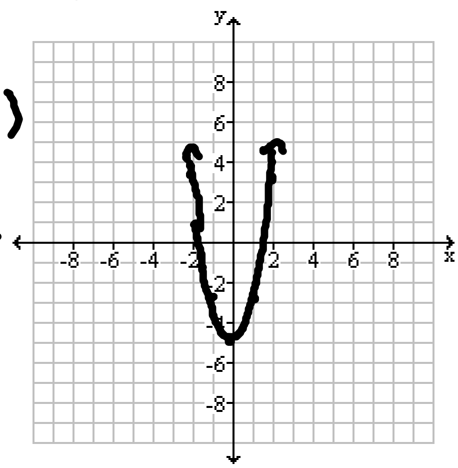
a) $y = 2x^2 - 5$

(0,0) → ( )

x	y
0	-5
1	-3
2	

$2(1)^2 - 5 = -3$

$2(2)^2 - 5 = 3$



Coordinates of the Vertex:

(0 , -5)

Equation of the axis of Symmetry:

x=0

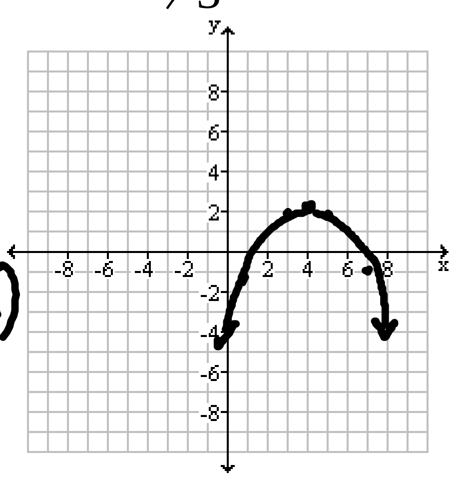
Domain & Range:

D: $x \in \mathbb{R}$ R: $\{y \mid y \geq -5, y \in \mathbb{R}\}$

$\boxed{\text{Minimum}}$ or Maximum Value:

(0 , -5)

b) $y = -\frac{1}{3}(x-4)^2 + 2$



(3,9) → (, 3)

Coordinates of the Vertex:

(4 , 2)

Equation of the axis of Symmetry:

x = 4

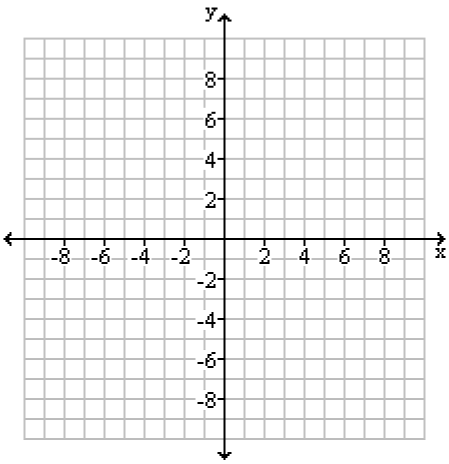
Domain & Range:

D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid y \leq 2, y \in \mathbb{R}\}$

Minimum or $\boxed{\text{Maximum}}$ Value:

(4 , 2)

c) $y = -3(x+7)^2 + 8$



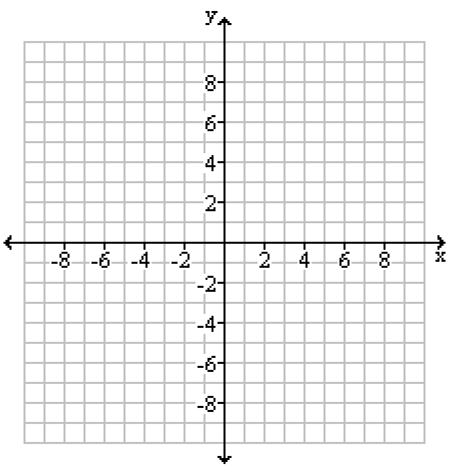
Coordinates of the Vertex:

Equation of the axis of Symmetry:

Domain & Range:

Minimum or Maximum Value:

d) $y = \frac{2}{5}(x-2)^2$



Coordinates of the Vertex:

Equation of the axis of Symmetry:

Domain & Range:

Minimum or Maximum Value:

Ex) Determine the equation for the parabola with its vertex at $(-1,\ 4)$ that goes through the point $(-2,\ 2)$.

x y

Set up a general eqn: $y = a(x+1)^2 + 4$

?

Use known point to solve: $(-2,2)$

$2 = a(-2+1)^2 + 4$

$2 = a(-1)^2 + 4$

$2 = a(1) + 4$

$2 = a + 4$

$2 - 4 = a + \cancel{4} - \cancel{4}$

$\boxed{-2 = a}$

$\boxed{y = -2(x+1)^2 + 4}$

Ex) Determine the equation of the parabola that goes through the point $(14,\ 22)$ and has its vertex at $(11,\ -14)$.

$$y = 4(x-11)^2 - 14$$

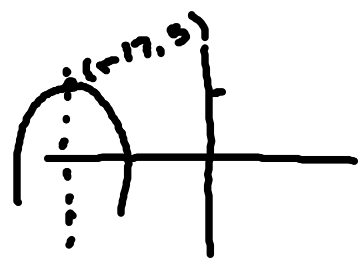
Ex) Determine the equation of the parabola that has an axis of symmetry given by $x = -17$, a range of $y \in R$ where $y \leq 5$, and goes through the point $(-24,\ -16)$.

Vertex: $(-17,\ 5)$

$$y = a(x+17)^2 + 5$$

Sub $(-24, -16)$: $-16 = a(-24+17)^2 + 5$

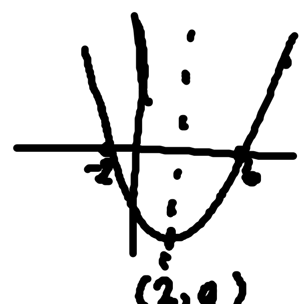
$$-16 = a(-7)^2 + 5$$
$$-16 = a(49) + 5$$
$$-16 - 5 = a(49)$$
$$\frac{-21}{49} = \frac{a(\cancel{49})}{\cancel{49}}$$
$$\frac{-21}{49} = a$$
$$\boxed{y = \frac{-21}{49}(x+17)^2 + 5}$$



Ex) Determine the equation of the parabola that passes through $(8,\ 5)$ and whose x-intercepts are -2 and 6.

$$\boxed{y = a(x-2)^2 + q}$$

Not helpful yet ∴ 2 unknowns



Sub $(8,5)$: $5 = a(8-2)^2 + q$

$$5 = a(36) + q$$
$$5 = 36a + q$$

Sub $(-2, 0)$.

$$0 = a(-2-2)^2 + q$$
$$0 = a(16) + q$$
$$0 = 16a + q$$

We have 2 eqns with 2 unknowns ∴ use substitution &/or elimination

$$\boxed{y = \frac{1}{4}(x-2)^2 - 4}$$

Ex) Describe the transformation required to turn the graph of $y = x^2$ into the graph of $y = -6(x+7)^2 - 10$.

open down

stretch (skinny)

move 7 left

move 10 down

Ex) A rocket is fired into the air. Its height is given by the equation $h(t) = -4.9(t-5)^2 + 124$, where h is the height in meters and t is the time in seconds.

a) Sketch the graph of this equation.

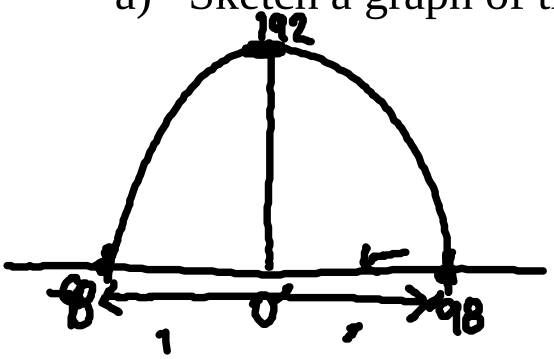
b) What is the maximum height reached by the rocket? and when does it reach its maximum height?

c) What was the height of the rocket when it was fired?

Ex) The stainless steel Gateway Arch in St. Louis, Missouri, has the shape of a **catenary** which is a curve that approximates a parabola. If the curve is graphed on a grid it can be modeled by the equation $h(d) = -0.02d^2 + 192$, where *d* is the horizontal distance from the centre of the arch and *h* is the height of the arch.

a) Sketch a graph of the shape of the arch.

(0, 192)



b) Find the height of the arch

192

c) Find the approximate width of the arch at its base.

98 + 98 = 196

d) Find the approximate height of the arch at a horizontal distance 15 m form one end.

$d = 83$

$98 - 15 = 83$

$h = -0.02d^2 + 192$

$= -0.02(83)^2 + 192$

$= -0.02(6889) + 192$

$= 54.22$ m

Now Try
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Completing the Square Part I:

When quadratics are written in the form

$$\boxed{y = a(x - p)^2 + q}$$

it is easy to graph them, this is known as **Standard Form**.

Quadratics can also be written in **General Form**

$y = Ax^2 + Bx + C$, where *A*,*B*, and *C* are Integers no fractions

no brackets

Standard Form:	$y = a(x - p)^2 + q$
General Form:	$y = Ax^2 + Bx + C$

In order to graph parabolas in general form we must first change it to standard form, this is done using a process called completing the square.

Factor the following.

$x^2 + 6x + 9$ $\qquad$ $x^2 - \boxed{10}x + 25$ $\qquad$ $x^2 + 22x + 121$

(x+3)(x+3) $\qquad$ (x-5)(x-5) $\qquad$ (x+11)(x+11)

What do you notice about the coefficient of the middle term and the constant? For all perfect squares if you take half the coefficient on the middle term and then square (mult by itself) it, we get our constant.

Ex) Find the value of c that makes each of the following a perfect square trinomial.

a) $x^2+14x+c$

$14 \div 2 = 7$

$7^2 = 49$

$x^2+14x+49$

$(x+7)^2$

b) $x^2-20x+c$

$-20 \div 2 = -10$

$(-10)^2 = 100$

$x^2-20x+100$

$(x-10)^2$

c) x^2+7x+c

$7 \div 2 = \frac{7}{2}$

$\left(\frac{7}{2}\right)^2 = \left(\frac{7}{2}\right)\left(\frac{7}{2}\right) = \frac{49}{4}$

$x^2+7x+\frac{49}{4}$

$(x+7/2)^2$

d) $a^2-\frac{5}{3}a+c$

$\left(-\frac{5}{3}\right) \div 2 = \frac{-5}{6}$

$\left(\frac{-5}{6}\right)^2 = \frac{25}{36}$

$a^2 - 5/3a + 25/36$

$(a-5/6)^2$

Ex) Convert the following to **Standard Form**.

→ Everything moves right except y

→ We want a bracket piece

a) $y=x^2+6x$

$y = \underbrace{x^2+6x+9}-9$

$= (x+3)^2-9$

3 left 9 down

b) $y=x^2-8x$

$(x-4)^2-16$

4 right 16 down

c) $y=x^2+10x+30$

$y = \underbrace{x^2+10x+25}-25+30$

$y = (x+5)^2+5$

5 left 5 up

d) $y=x^2+x-2$

e) $y = x^2 + 7x + 5$

$7 \div 2 = \frac{7}{2}$

$\left(\frac{7}{2}\right)^2 = \frac{49}{4}$

$$y = \underbrace{x^2 + 7x + \frac{49}{4}} - \frac{49}{4} + 5$$

$$y = \left(x + \frac{7}{2}\right)^2 - \frac{29}{4}$$

f) $x^2 - y + 12x - 11 = 0$

$$x^2 + 12x - 11 = y$$

$$y = x^2 + 12x - 11$$

$$y = \underbrace{x^2 + 12x + 36} - 36 - 11$$

$$y = (x+6)^2 - 47$$

g) $-3x^2 + 3y + 7x - 2 = 0$

$$\frac{\cancel{3}y}{\cancel{3}} = \frac{\cancel{3}x^2}{\cancel{3}} - \frac{7x}{3} + \frac{2}{3}$$

$$y = x^2 - \frac{7}{3}x + \frac{2}{3}$$

$\left(-\frac{7}{3}\right)\left(\frac{1}{2}\right) = -\frac{7}{6}$

$$y = \underbrace{x^2 - \frac{7}{3}x + \frac{49}{36}} - \frac{49}{36} + \frac{2}{3}$$

$\left(-\frac{7}{6}\right)^2 = \frac{49}{36}$

$$y = \left(x - \frac{7}{6}\right)^2 - \frac{25}{36}$$

Now Try
Worksheet

Completing the Square Part II:

So far we have only completed the square when the coefficient in front of the x^2 term is 1. If this value is something other than 1 we must first factor it out from the x^2 and the x terms.

Ex) Express the following in standard form. $y=a(x+p)^2+q$

a) $y=3x^2+18x+32$

$=3\left[x^2+6x+\frac{32}{3}\right]$ Factor 3 out of each term

$=3\left[x^2+6x\underbrace{+9-9}+\frac{32}{3}\right]$ Find constant to complete $\square$

$=3\left[(x+3)^2-9+\frac{32}{3}\right]$ Put in brackets.

b) $y=-2x^2+20x+17$

$=-2\left[x^2-10x-\frac{17}{2}\right] = -2\left[x^2-10x+25-25-\frac{17}{2}\right]$

$=-2\left[(x-5)^2-\frac{67}{2}\right]$

$= -2(x-5)^2+67$

Ans: $y=-2(x-5)^2+67$

c) $y=-x^2+8x-6$

Ans: $y=-(x-4)^2+10$

$y=-\left[x^2-8x+6\right]$

$=-\left[x^2-8x+16-16+6\right]$

$=-\left[(x-4)^2-10\right]$

$=-(x-4)^2+10$

d) $y = 5x^2 + 3x - 12$

$$= 5\left[x^2 + \frac{3}{5}x - \frac{12}{5}\right]$$

$$= 5\left[x^2 + \frac{3}{5}x + \frac{9}{100} \underbrace{- \frac{9}{100} - \frac{12}{5}}\right]$$

$$\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$$

$$\left(\frac{3}{10}\right)^2 = \frac{9}{100}$$

$$= 5\left[\left(x + \frac{3}{10}\right)^2 - \frac{249}{100}\right]$$

$$\boxed{= 5\left(x + \frac{3}{10}\right)^2 - \frac{249}{20}}$$

$$5\left(\frac{-249}{100}\right) = \frac{-249}{20}$$

e) $y = \frac{2}{3}x^2 - \frac{1}{5}x + \frac{4}{7}$

$$= \frac{2}{3}\left[x^2 - \frac{3}{10}x + \frac{6}{7}\right]$$

$$= \frac{2}{3}\left[x^2 - \frac{3}{10} + \frac{9}{400} - \frac{9}{400} + \frac{6}{7}\right]$$

$$= \frac{2}{3}\left[\left(x - \frac{3}{20}\right)^2 - \frac{9}{400} + \frac{6}{7}\right]$$

$$= \frac{2}{3}\left[\left(x - \frac{3}{20}\right)^2 + \frac{2337}{2800}\right]$$

$$\boxed{= \frac{2}{3}\left(x - \frac{3}{20}\right)^2 + \frac{779}{1400}}$$

Now Try
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Maximum & Minimum Problems:

Maximum and minimum word problems are ones that involve a quadratic equation and require us to find either a maximum or minimum situation.

Steps for Solving:

- Create a quadratic equation that describes the quality that you are finding the maximum or minimum of.
- Express your equation from the above step in **standard form** (complete the square).
- Determine the location of the vertex from the changed equation. This will tell you the maximum or minimum value as well as when it happens.

Ex) Find two numbers whose difference is 6 and whose product is a minimum.

x −

1st # = x

2nd # = y

Difference

$x-y=6$

$x-\cancel{y}+\cancel{y}=6+y$

$x-6=\cancel{6}-\cancel{6}+y$

$x-6=y$

Product

$\text{Min}=xy$

$\text{Min}=x(x-6)$

$=x^2-6x$

$=x^2-6x+9-9$

$=(x-3)^2-9$

Vertex: $(3,-9)$

x value

min

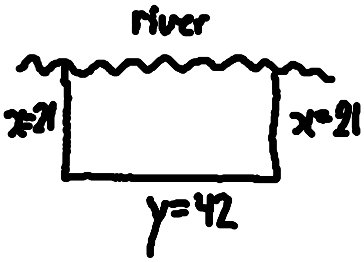
$x=3$

$y=x-6$

$=3-6$

$=-3$

Ex) A farmer wants to make a corral along a river. If he has 84 meters of fencing and if the river will act as one side of the corral, to what dimensions should the corral be built so the area contained within is a maximum?



Perimeter

$x+x+y = 84$

$2x+y=84$

$y=84-2x$

Area

Max Area $= L \times W$

$= xy$

$= x(84-2x)$

$= 84x-2x^2$

$= -2x^2+84x$

$= -2[x^2-42x+441-441]$

$= -2[(x-21)^2-441]$

$= -2(x-21)^2+882$

$y=84-2x$

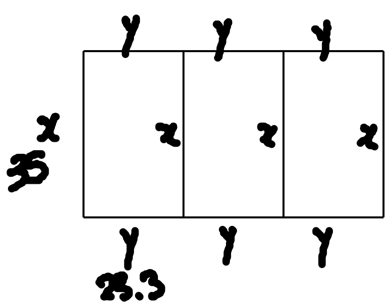
$=84-2(21)$

$=84-42$

$=42$

Vertex: $(21, 882)$
x max

Ex) 3 pens are to be created as shown below. If 280 m of fencing is available to create the pens, to what dimensions should they be built so that the area is a maximum?



Perimeter

$4x+6y=280$

$\frac{6y}{6} = \frac{280}{6} - \frac{4x}{6}$

$y = \frac{140}{3} - \frac{2}{3}x$

Area

Max Area $= L \times W$

$= x(3y)$

$= x\left(3\left(\frac{140}{3} - \frac{2}{3}x\right)\right)$

$= x(140-2x)$

$= 140x-2x^2$

$= -2x^2+140x$

$= -2[x^2-70x+1225-1225]$

$= -2[(x-35)^2-1225]$

$= -2(x-35)^2+2450$

$y = \frac{140}{3} - \frac{2}{3}(35)$

$= \frac{70}{3} \approx 23.3$

Vertex: $(35, 2450)$
x value Max Area

Ex) A hotel books comedians for a festival every year. Currently they charge \$28 per ticket and at this price sell all 500 tickets. The hotel is thinking of raising the ticket price. If they know that for every \$4 increase in the price of the ticket 20 fewer people will attend, what price should be charged per ticket to create a maximum profit?

x = # price increases

Profit = (Cost)(# tickets sold)

$= (28 + 4x)(500 - 20x)$

$= 14000 - 560x + 2000x - 80x^2$

$= -80x^2 + 1440x + 14000$

$= -80[x^2 - 18x - 175]$

$= -80[x^2 - 18x + 81 - 81 - 175]$

$= -80[(x-9)^2 - 256]$

$= -80(x-9)^2 + 20480$

Vertex: $(9, 20480)$

x value — max Profit

price increases = 9

Cost $= 28 + 4(9)$
$= 64$

sold $= 500 - 20(9)$
$= 320$

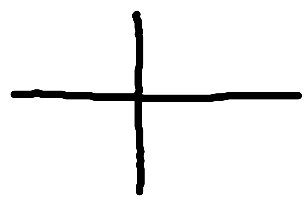
Now Try
Page 195 #18, 19, 20, 21
22, 23, 24

Quadratic Functions vs. Quadratic Equations

Quadratic Functions	Quadratic Equations
Are expressions relating two variables together. Can be represented by a graph.	Are equations that involve one variable that can be solved for.
Ex) $y = x^2 + 6x - 3$	Ex) $x^2 + 5x = -6$

Solving Quadratic Equations by Graphing:

To solve: $2x^2 + 20x + 42 = 0$ → Consider: $y = 2x^2 + 20x + 42$ when $y = 0$ → This means: Find the x-intercepts of $y = 2x^2 + 20x + 42$



We want to know what $x = ?$ so that when we sub the value back into our eqn, the left side = right side

Steps for Solving

- Bring all terms to one side of the equation (let it = 0).
- Graph the related function.
- Find the x-intercepts.

Ex) Solve $2x^2 + 20x = -42$

$$2x^2 + 20x + 42 = 0$$

$$\underbrace{2x^2 + 20x + 42}_{Y_1} = 0$$

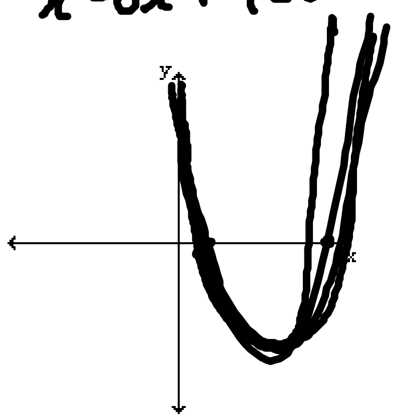
→ x-intercepts: $\boxed{2^{nd}}$ $\boxed{\text{Trace}}$ $\boxed{\text{Zero}}$

$$\boxed{\begin{array}{l} x = -7 \\ x = -3 \end{array}}$$

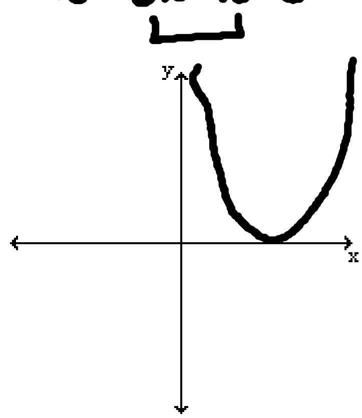
$2(-3)^2 + 20(-3) = -42$
$2(9) + 20(-3) = -42$
$18 - 60 = -42$
$-42 = -42$ ✓

Possible Outcomes when solving quadratic equations:

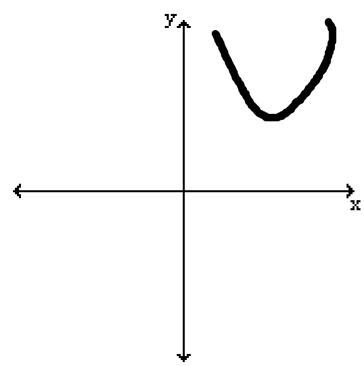
2 Real Roots	1 Real Root	No Real Roots
$x^2 - 8x + 9 = 0$	$x^2 - 8x + 16 = 0$	$x^2 - 8x + 20 = 0$



2 x-intercepts



1 x int. perfect square



doesn't cross x.

Ex) Solve the following by graphing a related function on your calculator. Round your answers to the nearest hundredth if necessary.

a) $x^2 + x = 6$

$$\underbrace{x^2 + x - 6}_{Y_1} = 0$$

$$\boxed{\begin{array}{l} x = -3 \\ x = 2 \end{array}}$$

b) $2x^2 = 3x + 34$

$$0 = \underbrace{-2x^2 + 3x + 34}_{Y_1}$$

$$\boxed{\begin{array}{l} x = -3.44 \\ x = 4\ 94 \end{array}}$$

c) $x^2 + 10x + 25 = 18$

$x = -9.24$

$x = -0.76$

d) $10 - 2x^2 = -8x + 18$

$x = 2$

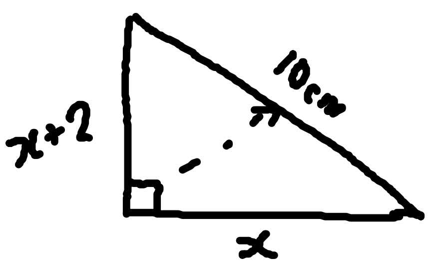
e) $x^2 + 19 = -8x$

No intercepts: no solution

f) $0.1(m+3)^2 = 1.6$

$0.1(m+3)^2 - 1.6 = 0$

Ex) The hypotenuse of a right triangle measures 10 cm. One leg of the triangle is 2 cm longer than the other. Find the lengths of the legs for the triangle.



$\boxed{x = 6\text{ cm} \qquad x+2 = 8\text{cm}}$

$\cancel{x = -8}$ Makes no sense

Pythagorean

$a^2 + b^2 = c^2$ hyp

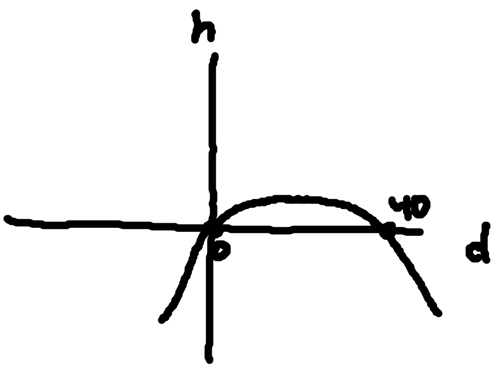
$a^2 + b^2 = 10^2$

$x^2 + (x+2)^2 = 100$

$x^2 + x^2 + 4x + 4 = 100$

$2x^2 + 4x - 96 = 0$

Ex) The function $h(d) = -0.025d^2 + d$ models the behavior of a soccer ball when it is kicked. *h* represents the height of the ball in metres and *d* represents the horizontal distance the ball has traveled in metres. Determine the horizontal distance the ball will travel when it hits the ground.



$\boxed{d = 40m}$

Now Try
Page 215 #3, 4, 5, 6, 8, 10
13, 14, 15, 16, 17

Solving Quadratic Equations by Factoring:

If $ab = 0$ then $a = 0$ or $b = 0$

$x^2 - 16 = 0$

$(x+4)(x-4) = 0$

Steps for solving

- Bring all terms to one side of the equations (let it = 0).
- Factor the equation.
- Let each factor = 0 and solve.

Ex) Solve the following by factoring.

a) $x^2 + 12x = -35$

$x^2 + 12x + 35 = 0$

	x	5
x	x^2	$5x$
7	$7x$	35

$(x+5)(x+7) = 0$

$x + 5 = 0 \qquad x + 7 = 0$

$\boxed{x = -5 \qquad x = -7}$

b) $3x^2 - 13x = 10$

$3x^2 - 13x - 10 = 0$

	x	-5
$3x$	$3x^2$	$-15x$
2	$+2x$	-10

Mult -30
Add -13

$(3x+2)(x-5) = 0$

$3x + 2 = 0 \qquad x - 5 = 0$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{-2}{3} \qquad \boxed{x = 5}$

$\boxed{x = -\frac{2}{3}}$

c) $x^2 - 25 = 0$ Difference of Squares

$(x - 5)(x + 5) = 0$

$x - 5 = 0 \qquad x + 5 = 0$

$\boxed{x = 5} \qquad \boxed{x = -5}$

d) $2x^2 + 3 = 5x + 1$

$2x^2 - 5x + 2 = 0$

	$2x$	-1
x	$2x^2$	$-1x$
-2	$-4x$	2

$(2x-1)(x-2) = 0$

$2x - 1 = 0 \qquad x - 2 = 0$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{1}{2} \qquad x = 2$

Mult 4
Add -5

e) $3x^2+5x=0$

$(x)(3x+5)=0$

$\boxed{x=0}$ $\quad 3x+5=0$

$\frac{\cancel{3}x}{\cancel{3}}=\frac{-5}{3}$

f) $\frac{2}{x+1}+\frac{5}{x-1}=-6$

$\frac{2\cancel{(x+1)}(x-1)}{\cancel{(x+1)}}+\frac{5(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}=-6(x+1)(x-1)$

$2(x-1)+5(x+1)=-6(x+1)(x-1)$

$2x-2+5x+5=-6(x^2-1)$

g) $x^2+4x=21$

$x^2+4x-21=0$

$(x+7)(x-3)=0$

$x+7=0 \quad x-3=0$

$\boxed{x=-7 \quad x=3}$

h) $4x^2-5x-21=0$

$(4x+7)(x-3)=0$

$4x+7=0 \quad x-3=0$

$\boxed{x=-\frac{7}{4} \quad x=3}$

i) $x^2+10x+24=0$

$(x+6)(x+4)=0$

$x+6=0 \quad x+4=0$

$\boxed{x=-6 \quad x=-4}$

j) $x^2+10=-25$

$x^2+35=0$

up 35

Cannot factor
∴ no solution.

k) $6x^2-5x+2=7+2x$

B of C

$6x^2-7x-5=0$

$(3x-5)(2x+1)=0$

$3x-5=0 \quad 2x+1=0$

$\boxed{x=\frac{5}{3} \quad x=-\frac{1}{2}}$

l) $a^2+22=-13a$

$a^2+13a+22=0$

$(a+11)(a+2)=0$

$a+11=0 \quad a+2=0$

$\boxed{a=-11 \quad a=-2}$

m) $x^2 - 64 = 0$

$(x+8)(x-8)=0$

$x+8=0 \qquad x-8=0$

$\boxed{x=-8 \qquad x=8}$

n) $2x - 40 = -x^2 + 8$

$x^2+2x-48=0$

$(x-6)(x+8)=0$

$x-6=0 \qquad x+8=0$

$\boxed{x=6 \qquad x=-8}$

o) $5a^2 = 52a - 20$

$5a^2-52a+20=0$

$(5a-2)(a-10)=0$

$5a-2=0 \qquad a-10=0$

$\boxed{a=\frac{2}{5} \qquad a=10}$

p) $x^2 - 16x + 64 = 0$

$(x-8)(x-8)=0$

$x-8=0$

$\boxed{x=8}$

q) $x^2 - 15 = -11$

$x^2-4=0$

$(x+2)(x-2)=0$

$x+2=0 \qquad x-2=0$

$\boxed{x=-2 \qquad x=2}$

r) $6x^2 + 20x = 0$

$2x(3x+10)=0$

$2x=0 \qquad 3x+10=0$

$\boxed{x=0 \qquad x=\frac{-10}{3}}$

s) $4x^2 - 20x + 25 = 0$

$(2x-5)(2x-5)=0$

$2x-5=0$

$\boxed{x=\frac{5}{2}}$

t) $3x^2 + 6x - 10 = 2x^2 + 6$

$3x^2-2x^2+6x-10-6=0$

$x^2+6x-16=0$

$(x+8)(x-2)=0$

$x+8=0 \qquad x-2=0$

$\boxed{x=-8 \qquad x=2}$

Ex) Determine a quadratic equation that has roots of $-\frac{1}{4}$ and 5.

$x = -\frac{1}{4}$

$4x = -1$

$4x + 1 = 0$

$x = 5$

$x - 5 = 0$

$(4x+1)(x-5) = 0$

$\boxed{4x^2 - 19x - 5 = 0}$

$x = ?$

Ex) Solve the following quadratic equation.

$$x(2x-3)+4(x+1)=2(3+2x)$$

$2x^2 - 3x + 4x + 4 = 6 + 4x$

$2x^2 - 3x + 4x + 4 - 6 - 4x = 0$

$2x^2 - 3x - 2 = 0$

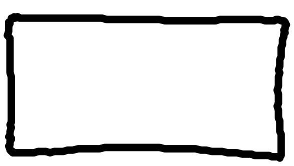
$(2x+1)(x-2) = 0$

	x	-2
$2x$	$2x^2$	$-4x$
1	$1x$	-2

$\rightarrow 2x + 1 = 0 \qquad x - 2 = 0$

$\boxed{x = -\frac{1}{2} \qquad x = 2}$

Ex) The length of a soccer pitch is 20 m less than twice its width. The area of the pitch is 6000 m^2. Find its dimensions.



$x = 60\text{m}$

$2x - 20$

$2(60) - 20$

$= 100\text{m}$

$L \times W = \text{Area}$

$(2x-20)(x) = 6000$

$2x^2 - 20x = 6000$

$2x^2 - 20x - 6000 = 0$

$2(x^2 - 10x - 3000) = 0$

$\rightarrow 2(x+50)(x-60) = 0$

$x + 50 = 0 \qquad x - 60 = 0$

$\cancel{x = -50} \qquad x = 60$

makes no sense

Now Try	
Page 229	#1, 2, 3, 5, 6, 7-10 (b, d, & f)
	11, 12, 15, 17, 24, 26

Solving Quadratic Equations by Completing the Square:

Ex) Solve the following:

$x^2 = 81$

$\sqrt{x^2} = \sqrt{81}$

$x = \pm 9$

$\boxed{x = 9}$ $\boxed{x = -9}$

$(x-3)^2 = 100$

$\sqrt{(x-3)^2} = \sqrt{100}$

$x - 3 = \pm 10$

$x - 3 = 10$ $\quad x - 3 = -10$

$\boxed{x = 13}$ $\quad \boxed{x = -13}$

Expressions of the Form $ax^2 + c = 0$ or $a(x-p)^2 + c = 0$ Can be Solved by:

- Isolate the term that is squared.
- Take the square root of both sides of the equation.
  *Remember when taking the square root we must consider both the positive and negative situation.
- Break equation into 2 separate equations (+'ve case and –'ve case), then solve each separately.
  *Leave answers in reduced radical form if necessary.

Ex) Solve the following using the square root principle.

a) $(x+3)^2 = 16$

$\sqrt{(x+3)^2} = \sqrt{16}$

$x + 3 = \pm 4$

$x + 3 = 4$ $\quad x + 3 = -4$

$\boxed{x = 1}$ $\quad \boxed{x = -7}$

b) $9x^2 - 21 = 0$

$\frac{9x^2}{9} = \frac{21}{9}$

$x^2 = \frac{21}{9}$

$\sqrt{x^2} = \sqrt{\frac{21}{9}}$

$x = \pm \frac{\sqrt{21}}{\sqrt{9}}$ $\quad \boxed{x = \pm \frac{\sqrt{21}}{3}}$

B
E
D
M
A
S

c) $(a+5)^2 - 6 = 30$

$(a+5)^2 = 36$

$\cancel{\sqrt{}}\cancel{(}a+5\cancel{)^2} = \sqrt{36}$

$a+5 = \pm 6$

$a+5 = 6$ $\quad$ $a+5 = -6$

$\boxed{a=1}$ $\quad$ $\boxed{a=-11}$

d) $\frac{\cancel{3}(x-2)^2}{\cancel{3}} = \frac{84}{3}$

$(x-2)^2 = 28$

$\cancel{\sqrt{(x-2)^2}} = \sqrt{28}$

$x-2 = \sqrt{28}$

$x-2 = \pm 2\sqrt{7}$

$\boxed{x = 2\sqrt{7} + 2}$ $\quad$ $\boxed{x = -2\sqrt{7} + 2}$

e) $\frac{\cancel{2}(2x-6)^2}{\cancel{2}} = \frac{88}{2}$

$(2x-6)^2 = 44$

$\sqrt{(2x-6)^2} = \sqrt{44}$

$2x-6 = \pm 2\sqrt{11}$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{\pm 2\sqrt{11} + 6}{2}$

$x = \sqrt{11} + 3$ $\quad$ $x = -\sqrt{11} + 3$

f) $4(y+1)^2 + 3 = 10$

$\frac{\cancel{4}(y+1)^2}{\cancel{4}} = \frac{7}{4}$

$(y+1)^2 = \frac{7}{4}$

$y+1 = \pm\sqrt{\frac{7}{4}}$

$y+1 = \pm\frac{\sqrt{7}}{2}$

$y = \frac{\sqrt{7}}{2} - 1$ $\quad$ $y = \frac{-\sqrt{7}}{2} - 1$

Equations in the Form $ax^2 + bx + c = 0$ Can be solved by:

- Bring all terms to one side of the equation (let it = 0)
- Complete the square putting the one side of the equation into standard form.
- Isolate the term that is squared.
- Take the square root of both sides of the equation
  *Remember when taking the square root we must consider both the positive and negative situation
- Break equation into 2 separate equations (+'ve case and –'ve case), then solve each separately.
  *Leave answers in reduced radical form if necessary.

Ex) Solve the following by completing the square.

a) $x^2+6x-27=0$

$x^2+6x+9-9-27=0$

$(x+3)^2-36=0$

$(x+3)^2=36$

$\sqrt{(x+3)^2}=\sqrt{36}$

$x+3=\pm 6$

$x+3=6$ $\boxed{x=3}$

$x+3=-6$ $\boxed{x=-9}$

b) $2x^2-x=10$

$2x^2-x-10=0$

$2\left[x^2-\frac{x}{2}-5\right]=0$

$2\left[x^2-\frac{x}{2}+\frac{1}{16}\underbrace{-\frac{1}{16}-5}\right]=0$

$2\left[\left(x-\frac{1}{4}\right)^2-\frac{81}{16}\right]=0$

$2(x-1/4)^2-\frac{81}{8}=0$

$\frac{2(x-1/4)^2}{2}=\frac{\frac{81}{8}}{2}$

c) $x^2+8x+11=0$

$x^2+8x+16\underbrace{-16+11}=0$

$(x+4)^2-5=0$

$(x+4)^2=5$

$\sqrt{(x+4)^2}=\sqrt{5}$

$x+4=\pm\sqrt{5}$

$x+4=\sqrt{5}$ $\boxed{x=\sqrt{5}-4}$

$x+4=-\sqrt{5}$ $\boxed{x=-\sqrt{5}-4}$

d) $2x^2-5x=1$

$2\left[x^2-\frac{5}{2}x-\frac{1}{2}\right]=0$

$2\left[x^2-\frac{5}{2}x+\frac{25}{16}\underbrace{-\frac{25}{16}-\frac{1}{2}}\right]=0$

$2\left[\left(x-\frac{5}{4}\right)^2-\frac{33}{16}\right]=0$

$2\left(x-\frac{5}{4}\right)^2-\frac{33}{8}=0$

$\frac{\cancel{2}\left(x-\frac{5}{4}\right)^2}{\cancel{2}}=\frac{\frac{33}{8}}{2}$

$\sqrt{\left(x-\frac{5}{4}\right)^2}=\sqrt{\frac{33}{16}}$

$x-\frac{5}{4}=\pm\frac{\sqrt{33}}{4}$

$x-\frac{5}{4}=\frac{\sqrt{33}}{4}$ $\boxed{x=\frac{\sqrt{33}+5}{4}}$

$x-\frac{5}{4}=-\frac{\sqrt{33}}{4}$ $\boxed{x=\frac{-\sqrt{33}+5}{4}}$

e) $4x^2 - 12x + 9 = 0$

$4[x^2 - 3x + \frac{9}{4}] = 0$

$4[(x - \frac{3}{2})^2] = 0$

$\frac{\cancel{4}(x - \frac{3}{2})^2}{\cancel{4}} = \frac{0}{4}$

$\sqrt{(x - \frac{3}{2})^2} = \sqrt{0}$

$x - \frac{3}{2} = 0$

$\boxed{x = \frac{3}{2}}$

f) $x^2 - 8x = -21$

$x^2 - 8x + 21 = 0$

$x^2 - 8x + 16 - \underbrace{16 + 21} = 0$

$(x - 4)^2 + 5 = 0$

$\sqrt{(x-4)^2} = \sqrt{-5}$

Cannot have − under radical

∴ no solution

g) $x^2 + 14x + 1 = 0$

$x^2 + 14x + 49 - 49 + 1 = 0$

$(x + 7)^2 - 48 = 0$

$(x+7)^2 = 48$

$\sqrt{(x+7)^2} = \sqrt{48}$

$x + 7 = \pm 4\sqrt{3}$

$x + 7 = 4\sqrt{3}$

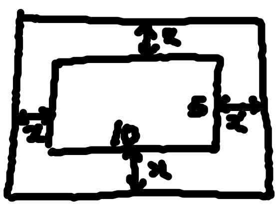
$\boxed{x = 4\sqrt{3} - 7}$

$x + 7 = -4\sqrt{3}$

$\boxed{x = -4\sqrt{3} - 7}$

h) $3x^2 - 11x = 10$

Ex) A picture that measures 10 cm by 5 cm is to be surrounded by a mat before being framed. The width of the mat is to be the same on all sides of the picture. The area of the mat is to be twice the area of the picture. What is the width of the mat?



Area Picture = $50 cm^2$

Area Mat = $100 cm^2$

$$(2x+10)(2x+5) - 50 = 100$$

$$(2x+10)(2x+5) = 150$$

$$4x^2 + 30x + 50 = 150$$

$$4x^2 + 30x - 100 = 0$$

$$4\left[x^2 + \frac{30}{4}x - 25\right] = 0$$

$$4\left[x^2 + \frac{30}{4}x + \frac{225}{16} \underbrace{- \frac{225}{16} - 25}\right] = 0$$

$$4\left[\left(x + \frac{30}{8}\right)^2 - \frac{625}{16}\right] = 0$$

$$4\left(x + \frac{30}{8}\right)^2 - \frac{625}{4} = 0$$

$$\frac{\cancel{4}\left(x + \frac{30}{8}\right)^2}{\cancel{4}} = \frac{\frac{625}{4}}{4}$$

$$\sqrt{\left(x + \frac{30}{8}\right)^2} = \sqrt{\frac{625}{16}}$$

$$x + \frac{30}{8} = \pm\frac{25}{4}$$

$$x + \frac{30}{8} = \frac{25}{4} \qquad\qquad x + \frac{30}{8} = -\frac{25}{4}$$

$$x = \frac{5}{2} \qquad\qquad \cancel{x = -10}$$

$$\boxed{= 2.5 \text{ cm}} \qquad\qquad \text{Makes no sense}$$

Now Try
Page 240 #1, 2, 5, 6, 7, 8, 13, 19

Solving Quadratic Equations using the Quadratic Formula:

Complete the square and then solve the following:

$$\boxed{ax^2 + bx + c = 0}$$

$$a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] = 0$$

$$a\left[x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a}\right] = 0$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{4ac}{4a^2}\right] = 0$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}\right] = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a} = 0$$

Solve for x.

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$

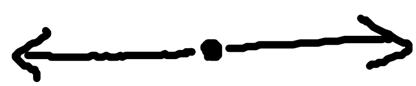
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^{\cancel{2}}} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$ Quadratic Formula



Vertex : $\boxed{x = \frac{-b}{2a}}$

to find y, sub x value into original eqn.

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex) Solve the following using the quadratic formula.

a) $x^2 + 3x - 28 = 0$

$a=1$
$b=3$
$c=-28$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-28)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{121}}{2}$$

$$= \frac{-3 \pm 11}{2}$$

$\boxed{x=4}$ $\boxed{x=-7}$

b) $3x^2 + 5x = 2 \rightarrow 3x^2 + 5x - 2 = 0$

$a=3$ $b=5$ $c=-2$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{49}}{6}$$

$$= \frac{-5 \pm 7}{6} \rightarrow x = \frac{-5+7}{6} \text{ or } x = \frac{-5-7}{6}$$

$\boxed{x = \frac{1}{3} \qquad x = -2}$

c) $x^2 - 2x - 1 = 0$

$a=1$ $b=-2$ $c=-1$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{8}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2}$$

$$= 1 \pm \sqrt{2}$$

$\boxed{x = 1+\sqrt{2} \qquad x = 1-\sqrt{2}}$

d) $\dfrac{2}{x+1} + \dfrac{3}{x-2} = \dfrac{2x^2}{x^2 - x - 2}$

$$\frac{2\cancel{(x+1)}(x-2)}{\cancel{(x+1)}} + \frac{3(x+1)\cancel{(x-2)}}{\cancel{(x-2)}} = \frac{2x^2\cancel{(x+1)(x-2)}}{\cancel{(x+1)(x-2)}}$$

$2x - 4 + 3x + 3 = 2x^2$

$-2x^2 + 5x - 1 = 0$ $a=-2$ $b=5$ $c=-1$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(-2)(-1)}}{2(-2)}$$

$$= \frac{-5 \pm \sqrt{17}}{-4}$$

$\boxed{x = \frac{5 + \sqrt{17}}{4}}$ $\boxed{x = \frac{5 - \sqrt{17}}{4}}$

Ex) Lindsay travelled from Calgary to Spokane, a distance of 720 km. On the return trip her average speed was 10 km/h faster. If the total driving time was 17 hours, what was Lindsay's average speed from Calgary to Spokane?

$d = vt$

$$\boxed{t = \frac{d}{v}}$$

Time to + Time back = 17

$$\frac{720}{x} + \frac{720}{x+10} = 17$$

$$\frac{720(\cancel{x})(x+10)}{\cancel{x}} + \frac{720(x)\cancel{(x+10)}}{\cancel{x+10}} = 17(x)(x+10)$$

$$720x + 7200 + 720x = 17(x^2 + 10x)$$

$$1440x + 7200 = 17x^2 + 170x$$

$$0 = 17x^2 + 170x - 1440x - 7200$$

$$0 = \underset{a}{17}x^2 - \underset{b}{1270}x - \underset{c}{7200}$$

$$x = \frac{1270 \pm \sqrt{(-1270)^2 - 4(17)(-7200)}}{2(17)}$$

$\vdots$

$\boxed{x = 80 \text{ km/h}}$ or $\cancel{x = -\frac{90}{17} \text{ km/h}}$ ← Makes no sense

The Discriminant:

The Discriminant is the part of the quadratic formula that is inside the square root.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \longrightarrow \boxed{b^2-4ac}$$

- If $b^2-4ac>0$ then, 2 solutions/2 roots/2 intercepts
- If $b^2-4ac=0$ then, 1 solution
- If $b^2-4ac<0$ then, no solution

Ex) Determine the nature of the roots for the following.
(Determine how many answers the following will have.)

a) $x^2-x-5=0$

$a=1$
$b=-1$
$c=-5$

b^2-4ac
$=(-1)^2-4(1)(-5)$
$=1+20$
$=21>0 \quad \therefore \boxed{2 \text{ solutions}}$

b) $x^2-8x+16=0$

$a=1$
$b=-8$
$c=16$

b^2-4ac
$=(-8)^2-4(1)(16)$
$=64-64$
$=0 \quad \therefore \boxed{1 \text{ solution}}$

c) $3x^2 - 11x + 12 = 0$

$a = 3$
$b = -11$
$c = 12$

$b^2 - 4ac$
$= (-11)^2 - 4(3)(12)$
$= 121 - 144$
$= -23 < 0$
$\boxed{\therefore \text{ no solutions}}$

d) $2x^2 + 3x = 4 \rightarrow 2x^2 + 3x - 4 = 0$

$a = 2$
$b = 3$
$c = -4$

$b^2 - 4ac$
$= (3)^2 - 4(2)(-4)$
$= 9 + 32$
$= 41 > 0$
$\boxed{\therefore \ 2 \text{ solutions}}$

Ex) Find the value of k so that $-2x^2 + 4x + k = 0$ has no real roots.

c value

discriminant < 0

$a = -2$
$b = 4$
$c = k$

$b^2 - 4ac < 0$

$4^2 - 4(-2)(k) < 0$

$16 + 8k < 0$

$\frac{\cancel{8}k}{\cancel{8}} < \frac{-16}{8}$

$\boxed{k < -2}$

That's it! Good Luck ☺

Now Try
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